# Nick Nesbitt\* Bolzano's Badiou<sup>1</sup>

Alain Badiou never mentions the pioneering, long-overlooked Czech-German logician Bernard Bolzano in the three volumes of *Being and Event*. In fact, his name only appears in passing, to my knowledge, on two occasions in Badiou's oeuvre: once in *Number and Numbers*, in a list of the modern founders of *the thought of number*, and once, in a passing reference to Bolzano's pioneering formalisation of the concept of the infinite in *Paradoxes of the Infinite*, in Badiou's 1994–95 seminar on Lacan.<sup>2</sup> Badiou has, moreover, admitted that his knowledge of Bolzano's work is in fact limited and largely second hand.<sup>3</sup> In what follows, I wish, briefly and in a very preliminary sense, to indicate a few of the ways Bolzano's thought in fact founds many of the essential categories and critiques developed throughout Badiou's oeuvre.

Badiou's neglect of Bolzano's thought is hardly surprising, since the philosopher's pioneering and foundational work, in set theory, in the critique of post-Kantian Idealism and intuitionism, in the semantic formalization of mathematics and logic, in the formal nature of axiomatisation, his precocious articulation of a realist, mathematics-based platonism a century before Albert Lautman's "transplatonism," and in many other fields, remained little acknowledged

<sup>&</sup>lt;sup>1</sup> The research and work on this study was supported by the Czech Science Foundation (GAČR) within the project (GA 19-20319S) "From Bolzano to Badiou."

<sup>&</sup>lt;sup>2</sup> "Les noms de cette première modernité [de la pensée du nombre] ne sont pas Proust et Joyce, ce sont Bolzano, Frege, Cantor, Dedekind, Peano." Alain Badiou, *Le Nombre et les nombres*, Editions du Seuil, Paris 1990. p. 24. "Après que l'infini eut reçu dans la mathématique un statut clair, grâce à Bolzano, Weierstrass et Cantor, il cesse de jouer un rôle dans l'argumentation philosophique." Alain Badiou, *Le Séminaire – Lacan*, Fayard, Paris 2013 pp. 256–257. In English: Alain Badiou, *Number and Numbers*, Robin Mackay, trans., Polity Press, Cambridge 2008. Alain Badiou, *Lacan: Antiphilosophy 3*, trans. Kenneth Reinhard and Susan Spitzer, Columbia University Press, New York 2018. See also Bernard Bolzano, *Paradoxes of the Infinite*, Routledge, New York 1950 [1851].

<sup>&</sup>lt;sup>3</sup> Personal communication, New York, 10.18.17.

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and even less studied until quite recently.<sup>4</sup> As late as 1993, Jacques Bouveresse could still decry this "historical injustice" done to "the most gifted and original adversary of German Idealism."5 Decades before Frege, Husserl, Cantor, Tarski, and Gödel, Bolzano founded or made possible many of the crucial discoveries of modern analytic philosophy and set theory, innovations for which the former would become famous. Following the prohibition of his publications and his early retirement to the Czech countryside, Bolzano's discoveries remained overlooked after his death in 1848, and thus the breakthroughs of his major works Paradoxes of the Infinite and Theory of Science were only belatedly recognized by Cantor and famously celebrated by Husserl in the Philosophical Investigations.<sup>6</sup> Only in recent years have Bolzano's contributions to philosophy begun to garner the recognition they deserve. Moreover, Bolzano's conservative moralistic pronouncements have nothing of the political daring of Spinoza or Cavaillès and Lautman-with whom he nonetheless shares many theoretical points of agreement-nor, to be sure, with Badiou's many developments of and fidelity to the idea of communism, and it is therefore even less surprising that Badiou should never have engaged with Bolzano's thought.7

## Bolzano's Badiou

That said, Bolzano's vast and still underexplored body of work announces Badiou's thought in a series of crucial dimensions. Here, I wish only briefly indicate

58

<sup>&</sup>lt;sup>4</sup> Badiou, *Being and Event*, p. 12.

<sup>&</sup>lt;sup>5</sup> Jacques Bouveresse, "Préface," in Jacques Laz, *Bolzano, critique de Kant,* Vrin, Paris 1993, p. iv.

<sup>&</sup>lt;sup>6</sup> "Bernhard Bolzano's *Wissenschaftslehre*, published in 1837, a work which, in its treatment of the logical 'theory of elements', far surpasses everything that world-literature has to offer in the way of a systematic sketch of logic." Edmund Husserl, *Philosophical Investigations*, Routledge, New York 2001 [1900], p. 68. See Bernard Bolzano, *Theory of Science*, Oxford University Press, Oxford 2014; *Théorie de la science*, *I-II*, Gallimard, Paris 2011.

On Bolzano's life and his (in my judgment) relatively banal and conservative moral, political, and aesthetic philosophy, which I will not address here, see the biographical information in Paul Rusnock and Jan Šebestík, *Bernard Bolzano: His Life and Work*, Oxford University Press, Oxford, 2019. It should be noted, however, that Bolzano publicly articulated as radical a critique of Viennese militarism as was perhaps possible in his Austro-Hungarian milieu, and it was this in particular that led to the banning of his publications and his forced early retirement from Charles University. On Badiou's "Idea of Communism," see Alain Badiou, *The Communist Hypothesis*, Verso, New York 2015.

some of the most evident examples of this relation, each of which remains to be developed comprehensively:

1. Bolzano's thought remains the most original and decisive critique of post-Kantian Idealism in the first half of the nineteenth century. While Badiou cannot be said to reject Hegelian dialectical modes of thought entirely, and in fact has returned repeatedly to interrogate their modalities, it is arguably Bolzano who initiates a tendency in European philosophy to supplement and complete philosophical investigations with apodictic demonstrations formulated in the precise, emphatically un-Hegelian mathematical terms of set-based theory. This mode of philosophical demonstration culminates in Badiou's methodological apparatus deployed throughout the three volumes of *Being and Event*. While Theory of Science will reiterate and refine the terms of Bolzano's initial critique of post-Kantian Idealism, Jacques Laz has shown that Bolzano's 1810 Beiträge zu einer begründeteren Darstellung der Mathematik [Contributions to an Exposition of Mathematics on a Firmer Basis], written when Bolzano was only twenty-nine, already sets forth the principal propositions of his thought.<sup>8</sup> Key among these is his systematic critique of Kantian philosophy, attacked at its root via what Bolzano shows to be the contradictory nature of Kant's claims for an *a priori* intuition that would ground the entire project of the Critique of Pure Reason.9 While the extraordinary brevity of the Appendix to Bolzano's Contributions ("The Kantian Doctrine of the Construction of Concepts by Intuitions") articulates its powerful critique in a mere eleven dense and methodically parsed paragraphs,<sup>10</sup> elsewhere Bolzano decries more generally the "love of imagistic language," lack of expressive precision, and reliance upon "analogies, paradoxes, and tautologies" dominant in the Schellingian and Hegelian thought of the age.<sup>11</sup> While Badiou's often highly imagistic and even poetic turns of phrase are decidedly unlike Bolzano's generally dry prose, these natural language excurses are systematically complemented, in Being and Event, by abstract formalizations that seek to produce purely objective statements on the nature of the various concepts Badiou develops therein. Bolzano unequivocally condemns what he views as a catastrophic tendency of philosophy, "the essence of [which] con-

<sup>&</sup>lt;sup>8</sup> Bernard Bolzano, *Premiers écrits: Philosophie, logique mathématique*, Vrin, Paris 2010.

<sup>&</sup>lt;sup>9</sup> Immanuel Kant, *Critique of Pure Reason*, trans. Paul Guyer, Allen Wood, Cambridge University Press, Cambridge 1998.

<sup>&</sup>lt;sup>10</sup> See Bolzano, *Premiers écrits*.

<sup>&</sup>lt;sup>11</sup> Cited at Laz, *Bolzano*, p. 33.

sists in [...] playing with images and passing off the slightest superficial analogy between two objects as an identity."<sup>12</sup> The core of this limitation, Bolzano concludes, is that "the thinkers of our age do not feel themselves in the least subject to [...] the *rules of logic*, notably to the obligation always to state precisely and clearly of what one is speaking, in what sense one takes this or that word, and then to indicate from what *reasons* one affirms this or that thing."<sup>13</sup> Whether or not one judges this an accurate characterisation of Hegelian negative dialectical thought, Bolzano's critique proved decisively productive for his invention of what Jean Cavaillès would famously call, in his posthumous On Logic and the Theory of Science, a "philosophy of the concept" that Badiou has gone on to develop across the three volumes of Being and Event.<sup>14</sup> While Cavaillès celebrates, in On Logic, Bolzano's rigorous attention to the necessary modalities of adequate, apodictic demonstration, he nonetheless critiques the ahistorical nature of these conditions, to offer instead a historically developmental concept of adequate demonstration.<sup>15</sup> Badiou can be said in turn to have taken from Cavaillès' critique a positive notion of ontology in its intrinsic relation to science and to mathematics in particular as the adequate language of being as being.<sup>16</sup>

2. Bolzano's thought, from its initial formulation in the 1810 *Contributions* to the posthumous *Paradoxes of the Infinite*, anticipated by decades not only Dedekind and Cantor's definitions of infinite sets, but also Russell's paradox of the set of all sets, and Frege's definition of number as a set of concepts with isomorphic extension.<sup>17</sup> In 1816 Bolzano constructed a proof that is the first

<sup>15</sup> Hourya Benis Sinaceur has argued compellingly that Cavaillès' critique of Bolzano indicates a subterranean Hegelianism latent in Cavaillès' thought. Houya Benis Sinaceur, *Cavaillès*, Les Belles Lettres, Paris 2013, pp. 114–116.

60

<sup>&</sup>lt;sup>12</sup> Cited at Laz, *Bolzano*, pp. 32–33.

<sup>&</sup>lt;sup>13</sup> Cited at Laz, *Bolzano*, p. 32.

<sup>&</sup>lt;sup>14</sup> Jean Cavaillès, *Sur la logique et la théorie de la science*, Vrin, Paris 2008. Note that beginning with his critique of Fregean logicism in "Meditation 3" of *Being and Event*, Badiou decisively rejects the notion of logic as a purely syntactic operation: "Logic is not a formalization, a syntax, a linguistic apparatus. It is a mathematized description of possible mathematical universes, under the generic concept of Topos." Cited at Peter Hallward, *Badiou: A Subject to Truth*, University of Minnesota Press, Minneapolis 2003, p. 109. I will return to this point below, in reference to Bolzano's innovative formalization of axiomatic method.

<sup>&</sup>lt;sup>16</sup> Thanks to David Rabouin for clarifying this point.

<sup>&</sup>lt;sup>17</sup> Laz, *Bolzano*, p. 42. See also Jan Šebestík, "La classe universelle et l'auto-appartenance chez Bernard Bolzano," in *Mathematical journal of the seminar*, ed. P. Zernos, Athens 1986;

strictly conceptual formulation of the concept of continuity, a definition that crucially refuses all dependency upon psychologistic notions of intuition. This in itself constituted a powerful rejection of Kantian Idealism, which had judged the concept of continuity to be irreducible to conceptualization.<sup>18</sup> It is therefore all the more surprising that neither Badiou's 1984-1985 seminar L'infini: Aristote, Spinoza, Hegel nor Badiou's culminating, comprehensive statement on the nature of the infinite and human reason, The Immanence of Truths, contains a single mention of Bolzano's name.<sup>19</sup> Paradoxes of the Infinite directly influenced Dedekind and Cantor, and presents, along with *Theory of Science*, summaries of Bolzano's principal insights on mathematics and ontology. The concept of the infinite, Bolzano argues in *Paradoxes*, applies only to pluralities; as such, an object may be defined as infinite if it bears an attribute that indicates an infinite plurality. Bolzano furthermore offers a proof of the objective nature of the infinite as concept, from the proposition that there exist infinitely many truths en soi.<sup>20</sup> In this manner, Bolzano decisively rejects the Hegelian notion of qualitative infinity, while also distinguishing his *actual* concept of the infinite from a mere potentiality (as Cauchy argued), as well as from that of Spinoza, whose concept of the infinite Bolzano understands as the "infinite which is capable of no further increase."<sup>21</sup> Instead, Bolzano argues that various concepts of infinite sets - such as that of all points contained in the circumference of a circle, without having to count those elements sequentially, have actual, objective existence independent of their psychological cognition (except in cases in which the set in question includes that subjective cognition).<sup>22</sup> Bolzano also argues that there exist infinite sets of differing sizes, since one infinite set may logically be a subset of another, an argument from which, as Rusnock and Šebestík point out, Cantor's theory of transfinite cardinals begins: "namely, by defining two multitudes (finite or infinite) to be equinumerous if and only if there exists a bijection (a one-to-one map) between them."<sup>23</sup> This, however, precisely indicates

Rusnock and Šebestík, *Bolzano* pp. 533–540. On the influence of these thinkers upon Badiou's thought, see Hallward, *Badiou: A Subject to Truth*, Chapter 9, "Mathematics and Science."

<sup>&</sup>lt;sup>18</sup> Laz, *Bolzano*, p. 41; Rusnock and Šebestík, *Bolzano*, pp. 520–533.

<sup>&</sup>lt;sup>19</sup> Alain Badiou, *L'infini: Aristote, Spinoza, Hegel*, Fayard, Paris 2016; Alain Badiou, *L'immanence des vérités*, Fayard, Paris 2018.

<sup>&</sup>lt;sup>20</sup> Rusnock and Šebestík, *Bolzano*, pp. 533–534.

<sup>&</sup>lt;sup>21</sup> Cited at Rusnock and Šebestík, *Bolzano*, p. 534.

<sup>&</sup>lt;sup>22</sup> *Ibid.*, p. 535.

<sup>&</sup>lt;sup>23</sup> *Ibid.*, p. 536.

a "paradox of the infinite," since this definition contradicts the claim that the whole is greater than the part, and thus of unequal size. Though in the face of this contradiction, Bolzano steps back from the Cantorian conception of transfinite numbers – the assertion that, in the case of infinite sets, such a one-to-one mapping demonstrates that two sets may have the same number of elements – it has been suggested that Bolzano's conception of infinite sets might nonetheless constitute a distinctly non-Cantorian theory of the sizes of infinite sets.<sup>24</sup> Petr Vopěnka summarizes Bolzano's contribution to the Cantorian theory of the infinite as a veritable asubjective phenomenology:

Bolzano's explanation of how this or that form of the phenomenon of the infinite is produced is [foundational]. For him, these different forms are produced by different structures of the corresponding basic multitudes or, as we would say today, in the relational structures of the corresponding communities of objects. From a given community of objects we abstract its relational structure, which means that we replace properties of its members and their relations (what remains is their pure presence) by sets of objects that have such properties, relations by sets of ordered pairs, triples, etc., of objects that enter into the corresponding relations [...] and thus immediately find ourselves in the mathematics of the twentieth century. [...] Bolzano's instructions thus became the program of the set theory of the twentieth century. [...] No one followed it publicly, and no one appealed to it. Mathematicians did not even know Bolzano's words and, in spite of this, they obediently accomplished what those words commanded. Thus it is not exactly a program in the true meaning of the word, but rather a prophecy which was fulfilled, because it was founded on a clear and far-sighted vision of things to come.<sup>35</sup>

That Badiou has never to date engaged with Bolzano's theory of the infinite is certainly understandable for many of the reasons given above, but this genetic relation in the history of thought unquestionably remains a fruitful path for future research on Badiou's thought.

<sup>&</sup>lt;sup>24</sup> This has been argued by Paolo Mancosu, a conclusion Rusnock and Šebestík ultimately find unconvincing. See Rusnock and Šebestík, *Bolzano*, p. 537.

<sup>&</sup>lt;sup>25</sup> Petr Vopěnka, *Vyprávění o kráse novobarokní matematiky* [The Story of the Beauty of Neobaroque Mathematics], Práh, Prague 2004, p. 212, cited at Rusnock and Šebestík, *Bolzano*, p. 539.

3. Bolzano, decisively influenced on this count by Leibniz, is arguably the first modern philosopher to clearly define mathematics as the adequate language of ontology in the form of a *mathesis universalis* based upon predicate logic derived from Aristotle's Posterior Analytic.<sup>26</sup> Bolzano argues in the Contributions that philosophy is the science addressed to the question "what things are necessarily *real*," while mathematics, in contrast, addresses the question "What properties must things *necessarily* possess to be *possible*?"<sup>27</sup> While philosophy attempts to prove the reality of particular objects *a priori* and unconditionally, mathematics, in Bolzano's formulation, constitutes the *a priori* science of the set of universal laws to which all possible objects are subject.<sup>28</sup> Scientific method in general is for Bolzano coterminous with the logical rigor of mathematical method.<sup>29</sup> While for Bolzano philosophy seeks to deduce the real existence of things (analogous to Badiou's project to define an asubjective phenomenal logic in Logics of Worlds), mathematics applies its analysis, Bolzano argues, to the possible existence of all objects as governed by general laws. Bolzano can be said to announce Badiou's demonstration of the laws governing the phenomenal appearance of things in Logics of Worlds: mathematics, Bolzano affirmed, develops a general theory of forms, which he defined as "a science that treats of the general laws (forms) to which things must conform in their existence."<sup>30</sup> While for Bolzano this constitutes an ontological affirmation, Badiou will reject categorial logic as identical with being as such, to argue instead that while mathematics constitutes the adequate language of what is *dicible* (sayable) of being, a categorial logic offers the means to conceptualize an asubjective phenomenology of worlds.

4. Bolzano inaugurates the modern Platonist rationalist realism that would see its fullest development in Gödel and Lautman, a tendency that Badiou has polemically affirmed as crucial to his own thought.<sup>31</sup> Bolzano's *Contributions* already formulates in 1810 a philosophy of objective forms [*Formen*] and the sys-

<sup>&</sup>lt;sup>26</sup> On Leibniz's influence on Bolzano, see Laz, *Bolzano critique de Kant*, pp. 33–35; and on Bolzano's reconfiguration and critique of Aristotelean logic, see Laz, *Bolzano*, pp. 27–30.

<sup>&</sup>lt;sup>27</sup> Cited at Laz, *Bolzano critique de Kant*, p. 29.

<sup>&</sup>lt;sup>28</sup> Cited at Laz, *Bolzano*, p. 45.

<sup>&</sup>lt;sup>29</sup> Laz, *Bolzano*, pp. 46–48.

<sup>&</sup>lt;sup>30</sup> Cited at Rusnock and Šebestík, *Bolzano*, p. 417.

<sup>&</sup>lt;sup>31</sup> See for example Alain Badiou, *Plato's Republic: A Dialogue in Sixteen Chapters*, trans. Susan Spitzer and Kenneth Reinhard, Columbia University Press, New York 2013.

tematic connection of truths that defines its structure. This structure, he argues, follows an objective configuration, independent of subjective intuition and psychological certainty. "In the domain of truth," Bolzano writes in the *Contributions*, "that is to say in the set of all true judgements, there reigns a certain *objective connection*, independent of all contingent *subjective knowledge* that we may develop of it. [...] To present this objective connection of judgments, that is to say, to choose a set of judgments and to order them such that any inferred judgment is mentioned as such, seems to me the true goal of scientific exposition."<sup>32</sup> Such is the method Bolzano declares for a science *an sich*, one in which the objective connection of true judgements remains strictly independent of any subjective thought or feelings of certainty or doubt. Bolzano's project, which culminates in the *Theory of Science*, is nothing less than this demonstration of a coherent methodology, one that would develop for mathematical logic a conceptual clarity and definition independent of all psychologism and reliance upon intuition.

5. Badiou's rejection of Fregean logical grounding in favour of an axiomatic presentation, affirmed in Meditation 3 of Being and Event, marks a central moment in his theoretical intervention: "Axiomatisation," Badiou writes, "is not an artifice of exposition, but an intrinsic necessity. Being-multiple, if entrusted to natural language and to intuition alone, produces an undivided pseudo-presentation of consistency and inconsistency. [...] Axiomatisation is required such that the multiple, left to the implicitness of its counting rule, be delivered without concept, that is, without implying the being-of-the-one."33 While, as David Rabouin points out, Badiou's notion of axiomatisation draws upon Hilbert and Bourbaki, one might note that Bolzano already presents in the second section of the Contributions the first explicit model of axiomatisation, decisively rejecting Kantian intuitionism.<sup>34</sup> There, Bolzano does not proceed via a demonstration of the nature of the axiom, which would return precisely to the very logicism axiomatisation seeks to overcome (and for which Badiou takes Frege to task in both Being and Event and, in more detail, in Number and Numbers). The axiom, Bolzano argues in terms that decisively announce those of Badiou, is derived neither

<sup>&</sup>lt;sup>32</sup> Cited at Laz, *Bolzano*, p. 43.

<sup>&</sup>lt;sup>33</sup> Alain Badiou, *Being and Event*, trans. Oliver Feltham, Continuum, London 2005, p. 43, translation modified.

<sup>&</sup>lt;sup>34</sup> David Rabouin, personal communication.

through an intuition, nor even as a minimally and generally acceptable common notion (as with Marx's definition of capitalism as the general accumulation of commodities), which would rely on a psychological recognition and agreement. but is instead, he argues, indemonstrable, and objectively so. Bolzano argues that it is precisely and minimally the indemonstrability of an axiom, rather than its essential nature, that can in fact be proven. This minimal proof is merely the verification that allows axioms to found the subsequent propositions subject to apodictic demonstration. "Neither deduction, nor demonstration of the truth of a proposition," Jacques Laz writes, "the *Deductio* of an axiom is the exposition of its status as principal [statut de principe] in an objective sequence of connections between propositions. It is the operation by which are revealed the propositions that are the principals for other propositions."<sup>35</sup> Objective without being a logical demonstration of the truth of an axiom, the Deductio founds the effective conditions of demonstration, deducing only that a given proposition possesses an axiomatic character, in the sense that it cannot be analytically reduced into subsidiary components.36

6. Finally, and though this may be a less than obvious claim, it is my contention that Bolzano offers compelling conceptual resources to develop the structuralist analysis of what Marx called "social form": structuralist analysis, that is to say, in the quite specific sense in which Louis Althusser and Pierre Macherey developed it in *Reading Capital.*<sup>37</sup> Here, Bolzano's concerted critiques of intuitionism, psychologism, and empiricism, and his concept of propositions in themselves can be said to second and further develop the Spinozist critiques that Althusser, Rancière, Macherey, and Balibar deployed in their readings of Marx's *Capital.*<sup>38</sup> If Althusser and Macherey in particular looked back three hundred years prior to Spinoza in order to develop their critiques of Hegel and Hegelian Marxism, it is surely no less plausible to suggest that Bolzano, who as mentioned above developed the single most rigorous critique of Kantian and Hegelian Idealism prior to 1848, might offer compelling theoretical arguments to further develop this anti-Hegelian line of thought. Bolzano argued for an objective semantics governing not subjective, hermeneutic knowledge of objects, but their objec-

<sup>&</sup>lt;sup>35</sup> Laz, *Bolzano*, p. 55.

<sup>&</sup>lt;sup>36</sup> *Ibid.*, pp. 52–56.

<sup>&</sup>lt;sup>37</sup> Louis Althusser et al., *Reading Capital: The Complete Edition*, Verso, New York 2016.

<sup>&</sup>lt;sup>38</sup> See Warren Montag, Althusser and His Contemporaries: Philosophy's Perpetual War, Duke University Press, Durham 2013.

tive properties and relations. He inaugurates, this is to say, the affirmation that Badiou will formalize in 1988 as the governing imperative of *Being and Event*: that mathematics "writes that which, of being itself, is expressible [*dicible*]."<sup>39</sup> This, Bolzano argues, implies the independent existence of these concepts apart from conscious representation. Their meaning, he argues, is rigorously objective and independent from acts of judgment. In fact, I would willingly push this argument even further, to suggest that Bolzano can rightly be said to formulate crucial theoretical resources in the path leading to the Lacanian theory of the symbolic and real, above all perhaps via his realist, semantic critique of the Kantian thing in itself. As Badiou writes of Lacan's notion of the real,

Lacan is not a critic. To be sure, the real differs from reality, which attaches its regime to knowing. But Lacan immediately says: I don't mean to say the real is unknowable. I'm not a Kantian. [...] Although the real, as distinct from reality, is exempted from the knowable, which is the essence of reality, *the real nevertheless does not end up being the absolute unknowable but is instead exposed to being demonstrated*.<sup>40</sup>

Bolzano's asubjective order of propositions and representations, in a precise and limited sense analogous to what Lacan will call the symbolic order (in what Badiou calls Lacan's "hyperstructural axiomatic" phase of the 1950s), Bolzano argues, is eminently knowable through acts of human formalization and judgment, in contrast to Bolzano's anti-Kantian notion of the thing in itself as much as the Lacanian real.<sup>41</sup> While this objective order presents things as they are in what Bolzano calls the matter [*Stoff*] of a semantic, symbolic order, it is for Bolzano (unlike Kant) the (real) object of these representations and discursive judgements that remains inaccessible; the real, as Lacan famously stated, is the *impasse* of formalisation.<sup>42</sup> Or as Laz writes, for Bolzano, "we will never be able to grasp the objects of our representation, but only their [objective] meaning through which we represent them."<sup>43</sup>

<sup>&</sup>lt;sup>39</sup> Badiou, *Being and Event*, p. 5, translation modified.

<sup>&</sup>lt;sup>40</sup> Badiou, *Lacan*, p. 151.

<sup>&</sup>lt;sup>41</sup> *Ibid.*, p. 237.

<sup>&</sup>lt;sup>42</sup> Badiou, *Being and Event*, p. 5.

<sup>&</sup>lt;sup>43</sup> Laz, *Bolzano*, pp. 121–122.

To suggest a Bolzanian reading of Badiou along the lines that I am suggesting here is surely no more implausible than was Pierre Macherey's influential reading of Spinoza's Hegel.<sup>44</sup> It is to articulate a transversal relation; unlike that which Macherey articulates, however, in Badiou's case, there is no obscure disavowal on his part of a hidden proximity to Bolzano's historically prior thought, but rather a complex field of relations and implications that remains to be developed and articulated, an investigation that Badiou himself might be the first to welcome.

While I have here tried only to suggest a few of these possible paths of research, it seems to me that Bolzano's thought is no mere antiquarian moment in a history of axiomatic philosophies, philosophies that hold mathematics to constitute the adequate language of being. At least as promising, for example, would be to further concretize the anti-Hegelian, objective dimensions of apodictic demonstration that Althusser, Macherey and Badiou himself have argued govern not only much of their own thought, but above all, the critical projects of Marx and Lacan. Such a project might remain faithful to the imperative that Badiou has argued governs his philosophical project as a whole: "To legitimate the claim that a truth can be absolute, while also a localized construction, [...] eternal, while belonging to the time of this world [... and] a-subjective, while demanding, to be grasped, a subjective incorporation."<sup>45</sup> To place Badiou's philosophy of being and event in dialogue with that of Bolzano in this manner would imply the exploration of truths developed and demonstrated in suspension, truths articulated between these two figures, in the diffraction of their mutual demonstrations, as both the critique and proof of philosophy itself in the set that constitutes its own historicity.

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<sup>&</sup>lt;sup>44</sup> Pierre Macherey, *Hegel or Spinoza*, University of Minnesota Press, Minneapolis 2011 [1979].

<sup>&</sup>lt;sup>45</sup> Badiou, *L'Immanence des vérités*, p. 13.

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