Development of logical form

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In this paper, I would like to point out the problems of the presently reigning mathematico-functional concept of the logical form of sentences, which presents itself as the final answer to the question of true logical form of sentences and, with this, the final basic scheme of logic. I am of the opinion that the present conception of the logical form of sentences is also a historical result, which in many ways surpasses and encompasses all former concepts in the history of logic, but which is not the only and absolute logical form of sentences, but also of logic itself. It therefore contains some immanent limitations which are, in my opinion, linked mainly to the »functional« concept of the sentence, the elementary predicative sentence, which is the foundation for all other sentence structures.

Logic, with its use of letters for marking variables, which represent arbitrary actual terms, received the possibility of simultaneously treating a whole class of logically identical deductions. Mathematics also analogously received the means for executing general solutions for a whole class of related tasks. For instance, in geometry equations, make it possible to treat the properties of the most general classes of geometrically similar figures.

The link between logic and mathematics is even more tight in the notion of deduction, as both branches are strict deductive sciences, and the ideal of deduction is most surely the axiomatic system. As Lukasiewicz showed, Aristotle's syllogistic can be partly presented as an axiomatic system, constructed from the modi of the so-called »first figures« of the syllogistic as axioms and additional rules of substitution and transformation of individual statements (J. Lukasiewicz, *Aristotle's Syllogistic,* 1951).

Aristotle's notion of the logical form of a sentence is entirely linked to the subject-predicate structure of sentences. It expresses the constant core of the sentence that remains if we exclude the term for the subject and the predicate. To this we must add that Aristotle usually deals with quantified, universal or particular sentences, since sentences on the individual are scientifically irrelevant for him, as they only deal with pure contingencies.

Aristotle's conception of the logical form of a sentence was strongly influenced by his ontology, where the basic ontological fact is the individual substance with its essential or contingent properties. The unity of substance and its properties is expressed by the simple predicative sentence »a is P«, where a is the name of an individual and P a name of its property.

This is not only Aristotle's discovery, as Plato already defined (Platon, *Sophistes,* 262) the predicative sentence as the basic expression of truth, where the »name« *(onomata)* and verb *(remata)* link through a copula. But, for Aristotle, something else was important in expressing a certain fact besides the sentence, namely, the cause of the fact, that is, the cause of the so-linked substance and its properties.

Aristotle's conception of the logical form of deduction attempted to determine the logical place for cause in the syllogism. The basic logical form of the syllogism thus had to correspond to his understanding of cause. And here is the essence of Aristotle's theory of the syllogistic. We could say that Aristotle reached the syllogistic through his search for logical categorization of causes understood as mediators between the subject term and predicate term (Aristotle, *Anal. Post.* II, 89b,90a).

The paradigm of Aristotle's concept of cause was *causa formalis,* which best expresses the essential link of the individual to the general. *Causa formalis* is always something general, the essence of a substance. As stated by J. M. Blond, Aristotle's search for the reason »why something is« is actually reduced to the knowledge of »what a thing is« (»what a thing's nature is«) *(Anal. post.* 90a): »The search for the cause of a thing is actually not the aspiration for a link to another thing, but precisely the search for what it is itself«, (J. M. le Blond, *Logique et methode chez Aristote,* 1973, p. 100.)

Due to this ontological dependency of Aristotle's logic, we must not treat his logic with the attitude »to what extent propositional logic is present and to what extent predicative logic« is developed, because the syllogistic link of premises and conclusions are, for Aristotle, neither prepositional logic implications nor *modus ponens* derived from them (this was introduced by the stoics) or predicate logic. This actually concerns a mixture of both, a logical system that is antecedent to the division of propositional and predicate logic.

Aristotle's logic is thus still far from mathematical calculus or formalism, even far from the partial form of formalism that ancient algebra achieved. Only the use of variables applies to all of them. But while this was only an auxiliary measure for shortening the depiction of logical proofs of valid and invalid syllogisms, in Greek algebra they already served as an operational tool for solving equations (e.g. Diophant and later Archimedes) (M. Kline, *Mathematical Thought from Ancient to Modern Times,* 1972, pp. 139-144). In stoic logic, the next stage in the development of logic, Greek logicians used variables to denote the constant and essential in the form of a logical deduction. But for their foundation they took pure logical »relations« of sentences and the deduction of sentences from sentences, without any ontological support of the sentence or the deduction.

In this sense, their formalism is nearer to pure logic than Aristotle's and, thus, also closer to the logical essence of deduction as a propositional logic

operation. But their logic remains limited because it has once again been confined to sentence logic only. It seems that they accepted Aristotle's syllogistic as valid, but they held their own logic to be more fundamental than syllogistic logic, although it is not known whether they had systematically linked both (W. and M. Kleene, The Development of Logic, 1971, pp. 175).

But in stoic logic, the deduction also was a representative of the very »strong« kind of causality (the causal determinism of all events).

In both (stoic and Aristotle's logic) examples logical form also indicates the basic philosophical premises from which logic proceeds as well as the internal limits for both Aristotle's and stoic logic.

As with Aristotle, the central ontological unit for Leibniz, who attempted to reach a new step in the formal representation of sentences and in the mathematical formalisation of syllogism, was the individual substance (monad) and its attributes, where the basic issue was why a specific substance exists in the present state of the world. The cause was »unified« by Leibniz to such a degree that he does not separately state individual causes but conceives everything as an expression of the law of sufficient reason. Leibniz admits only *causa efficiens* and *causa finalis,* where both are in mutual harmony, but this harmony is »held in the grip« of God only, as the highest cause and reason of all existing (G. W. Leibniz, *Neue Abhandlung über den menschlichen Verstand,* 1926, pp. 480).

The limitations of the syllogistic were also possibly the reason that he could not realize his ideas, as he could never find a suitable calculus that would include all valid syllogisms and exclude all invalid ones, or he had to add a number of unclear operations without any logical sense. Nevertheless, in certain places he had an inkling of sentential logic when, for example, he discussed calculations containing only 0 and 1, the idea of which he derived from Chinese ideograms (Leibniz, *Fragmente der Logik,* 1960, pp. 23).

Only George Boole reached further than Leibniz, but more than 200 years later, in that he discovered the calculus or algebra of sentences. Besides this, he developed a special algebra of syllogisms. But Boole did not understand the meaning of his discovery of sentence logic, as he understood it as a species of a logical game; he developed serious logic in the syllogistic. The basic idea of this algebraic logic is that a specific, general, purely formal symbolic calculus of signs is established, to which we then assign various models of real operations. One of them can also be logical (there could be a number of them, for example, Boole's sentence calculus and extension calculus).

Due to this mainly algebraic method, Boole's logic represented only the »mathematization« of logic and not true mathematical logic. That is, logic and mathematics should be equally linked, without mathematics being applied only on previously derived logical formulations. Even Leibniz's logic can be

understood in this sense as a mathematizing logic, although he tried to give greater stress to specifically logical considerations.

True »mathematical logic« does not begin until Gottlob Frege. It was with him that a move in the conception of the logical form of the sentence came about, and it was shown how this was at the same time the embryo of mathematical operations and the logical analysis of the sentence. He was the first to come to such »mathematical« formulations of logic, which at least does not obviously introduce unnecessary symbols and operations, which only the calculus itself conjures up, without their being founded in logic proper.

Frege introduced three basic syntactical innovations into his logic. The first was a firmly principled propositional logic for all logics, while the second was the concept of logical form of the predicative sentence, constructed according to the model of mathematical functions; and the third was the concept of logical form of the quantified sentence. Then he linked his unique semantics of sense and meaning of linguistic expressions to this logical syntax, as well as his broadly founded and branched axiomatic predicative logical system (more exactly, first-order predicate logic with identity).

According to Frege, the simple predicative sentence without quantifiers represents an example of »completing a function«, namely, the so-called »predicative function«. This is, grammatically speaking, the predicate of the sentence together with the copula. The predicate itself is thus conceived explicitly as an incomplete expression; it is the recipe for constructing full sentences and is not a »part« of a sentence, which would exist besides the names of individuals and with which it would link into a sentence. But the name does step into the predicate in the way an argument steps into a mathematical function.

According to the mature concept of Frege, the function in the sentence maps the name, which enters the place of the argument, that is, the subject of the sentence, into a certain truth value, that is, Truth or Falsity, which Frege understands as two special logico-semantic entities (Truth and Falsity).

But in connection with the logical form of quantified sentences, Frege has warned already in »Begriffschrift« that the logical form of the sentence is something relative, because it is dependent on our context of understanding or context of understanding discourse Frege, e.g., states this example of a sentence:

»Hydrogen is lighter than Carbon Dioxide«, which we can interpret in a number of ways. For instance as an S-P sentence:

»Hydrogen is lighter than Carbon Dioxide.« Here, Hydrogen is the subject, and »is lighter than Carbon Dioxide« is the sentence predicate. Or »Hydrogen is lighter than Carbon Dioxide«, where »Carbon Dioxide« is the subject and »Hydrogen is lighter than« the predicate. Or, »Hydrogen is lighter than Carbon

Dioxide«, where »Hydrogen« and »Carbon Dioxide« are both arguments for the predicate (relation) »is lighter than« (G. Frege, *Begriffschrift,* 1964, pp. 15).

I try to improve this Fregean idea of the relativity of the logical form of a sentence on our context of understanding of sentence with the relativity of a sentence especially on the deductions, which we attempt to carry out with the sentence (the sentence may taken the role of a premise or a conclusion). We now attempt to give this opinion a new weight.

For each of these examples, we could state an example of deduction demanding just this form of sentence for its success.

For ex.: »No gaseous oxide is lighter than Carbon Dioxide.«

»Hydrogen is lighter than Carbon Dioxide.«

Therefore: »Hydrogen is not a gaseous oxide.«

Here, for the success of the deduction, only an ordinary syllogistic structure of premises is responsible, thus, for our sentence »S – P« (more exactly; Every S is P), where S is »Hydrogen« and P »is lighter than Carbon Dioxide«.

We can follow further this idea of the dependency of the logical form of a sentence from the »deductive context« of sentence in the other cases of deduction. We could e.g. also probably make a certain syllogism for the case when the subject is »Carbon Dioxide« and the rest the predicate (only that it would be more comfortable to write »Carbon Dioxide is heavier than Hydrogen«, which is in sense equal to the sentence »Hydrogen is lighter than Carbon Dioxide«).

But the deduction

»Hydrogen is lighter than Carbon Dioxide.«

»Carbon Dioxide is lighter than Nitrogen Dioxide.«

Therefore: »Hydrogen is lighter than Nitrogen Dioxide«,

despite the appearance of a syllogistic conclusion, does not concern a syllogism or S-P sentences, but a deduction concerning relations, which presupposes the so-called »transitivity« of the relation »is lighter than«. Thus the form of our sentence is relational.

Possibly the form of our sentence with a two-member relation seems to be a »complete« logical form of sentence, but this is only apparent, because in other contexts the sentence, e.g., could show itself as a case of more than a two-member relation or some other logical structure.

If, e.g., we consider that »is lighter than« means comparison of the specific weights of two elements and that this means that two elements have, at the same volume, same temperature and same pressure, two different weights, then our sentence actually means a five-member relation:

»At a given volume, temperature and pressure, the weight of Hydrogen is less than the weight of Carbon Dioxide.«

If we considered also what the expression »weight« means and how we check differences in weight, then we would undoubtedly get a very complicated sentence, whose form would be far from the simple diadic relation of two terms.

It is simple to continue this intuition so far that it also encompasses sentence logic. Namely, in cases with pure propositional logic deductions, only the sentence as such is important, without respect to the internal structure. In this case its »logical form« is the limiting case of form; it is only a bare sentence or perhaps the negation of a certain sentence.

Similarly, we could get inferences demanding explication of quantifiers, hidden in the sentence. Let us take the following deduction:

»Hydrogen is lighter than Carbon Dioxide«, thus
»One liter of Hydrogen is lighter than one liter of Carbon Dioxide«.

If we want to prove the validity of this simple deduction, we must reach for quantifiers in the hypothesis:

»For every x and for every y, if x is the volume of Hydrogen and y the volume of CO_2 and if x equals y, then the weight of x is less than the weight of y«.

If we add the following claim to this sentence: »We have 1 liter of Hydrogen and 1 liter of CO_2«, then, with this additional claim, we can, from the above hypothesis, reach the following conclusion: »A liter of Hydrogen is lighter than a liter of CO_2.«

Possibly we would have to reach even further than quantification if we studied, e.g., deduction in modal and other intentional contexts. Frege and the majority of modern logicians later avoided these contexts with the postulate of extensionality, but already a few simple cases show that we cannot stop with logical forms only at quantified sentences. The context of proving itself »provokes« new, previously unnoticed aspects of possible logical forms in the same sentence. Take, e.g., a certain modal context, the »real« modalities in which we do not ask ourselves whether they exist or not, but we observe only the possible deductions with them. Let as look at the following deduction:

»In mixtures of gas, the lighter gas necessarily separates first.«
»H_2 is lighter than CO_2.«
Therefore: »In the mixture of H_2 and CO_2, H_2 is separated first.«

But we know that in modal contexts, ordinary rules of existensionality do not apply, that is, we cannot simply install only existensionally equal terms (individual or universal) into the place of given terms, but only equal terms, otherwise we reach known paradoxes (W.v.O.Quine, *From a Logical Point of View,* 1963, pp. 43-44).

That is why we must write our sentence in this example also as a necessary sentence, thus: »H_2 is necessarily lighter than CO_2.« Thus the ordinary form of the sentence is joined also with the modal quantifier of judgment.

I think that I have shown enough with these examples to illustrate how the logical form of the sentence is formed in dependence on the deductive context of discourse. We see that the range of the logical form of the sentence reaches from the most elementary form, the bare variable for the sentence or the negation of the sentence, through various versions of predicative, relational and similar sentences, to quantified sentences and to modal and other intentional sentences. Here either extensions (supplements) or narrowings of antecedently given logical forms may occur in the transition from one context to another.

In the Fregean new conception of the logical form of quantified sentences lies also a new ontological role of the subject and the predicate in a sentence. In this new conception, the predicate of the sentence receives the active role, while the subject enters it only as a »filler« of empty places for arguments. In traditional conceptions, it was just the opposite: the subject was the ontological as well as the logical source of the sentence, thus also of predication.

This new conception without doubt brought about many entirely new logical discoveries, especially in mathematics. The new conception of the predicate and bound variables also allowed the discovery of various forms of variable binding, which corresponds to the role of pronouns in language. A more precise analysis of sentences and proof was made possible, especially those where relations appear. But this conception also has its limits because, as we saw before, we cannot say that a certain form of logical sentence structure represents its logical form for ever, in all contexts. The quantification formulation of universal and particular sentences is probably the most suitable for them, but it is a question of whether we can translate modal and other intentional contexts onto it. Here contemporary logic obviously breaks and forks into various currents.

If we are precise, these concepts do not flow smoothly even in the strict dimensions of extensional logic. Thus, e.g., numerous sentences with pronouns are not also examples of quantified sentences. But there, the traditional conception of S-P sentences works out well, which did not know quantification in our sense.

Let as take the following example:
»The woman with whom I saw you is beautiful.«

Contemporary interpretation would understand this sentence as a typical sentence using definite descriptions. In this case we would have the description: (That) »woman with whom I saw you.« But the difficulty here is that in the description itself the pronoun »with whom« appears again, that is, we refer to the same woman to whom we are attempting to give a description.

According to Russell, the above sentence should be an »existential sentence«:

»There exists a being, such that she is the »woman« with whom I saw you« and »she is only one« and »is beautiful«.

But this is not the end of the analysis, because a pronoun is once again in the description of the predicate itself, which obviously refers to the woman, the same, which is also »that being« to whom the link with the quantifier already refers.

Possibly this could pass:

»There is a $being_1$ and a $being_2$, such, that '$being_1$ is a women' and 'I saw you with $being_2$' and 'for every $being_3$, if I saw you with $being_3$, then ($being_2$ = $being_3$)' and ($being_1$ = $being_2$) and '$being_1$ is beautiful'.«

More formally written:

(x)(y) (x is a woman & I saw you with y & (z) (I saw you with z → (z = y)) & (x = y & x is beautiful)). (The symbol »(..)« means the universal quantifier »for each..«, the symbol »&« means conjunction, the symbol »→« means implication.)

It is obvious that in some way we have become totally lost with this analysis, because all of a sudden, instead of one object to which we are referring, we have two, or, at least, two existential quantifications that independently range over the domain of discourse. And this probably is not equal to the primary logical sense of the sentence, although a modern logician would state that this is the price of exactness. But traditional logic would interpret this sentence simply and clearly, as it would have permanently retained the »link« of reference to the subject, that is, the specific woman.

There are many such examples, and even Quine mentioned somewhere that modern logic is incapacitated before them. The difference is in the fact that modern logic interprets definite descriptions as: »that (woman with whom I saw you)« while the traditional would interpret this as »that woman (with whom I saw you),« which at least seems more natural to the eye (W.W. Quine, *Logic as a Source of Syntactical Insights,* 1976). (G. Englebretsen, *Notes on Quine's Syntactical Insights,* 1984, p. 154).

Besides the stated comments, something more could be said, especially about Frege's introduction of functions instead of predicates. Frege is spinning around in a circle here. On the one hand, function means a certain rule of mapping one class into another; in the case of predicate functions this would be a mapping from a class of possible arguments (of all »objects«) into the class of truth-values.

On the other hand, each class is defined only with a specific predicate or, should we say, function (at least according to Frege). Besides this, every mathematical function actually means a logical rule, which states when two values may be placed into a certain relation (relation of argument and value of

the function). Thus, the predicative is already contained here. That is why we cannot take this same mathematical procedure for »ing« predication, as it is obviously a false circle, explaining nothing.

The conclusion here is that Frege's functional interpretation of the logical form of the predicative sentence is not universal and the only one possible, but that it is only a good analogy for a series of logical forms of sentences in specific contexts of discourse and, even better, in certain contexts of proof. And these are above all the fields of mathematical and natural sciences, where the regulation of existensionality may be quite broadly established. But, as we saw, even there it is not valid for all examples.

This applies less to social sciences, where intentional contexts of propositional orientation appear, and even less in daily »ordinary language«. The revealing of sentence form can thus be an ever-so-innovative venture, and we cannot definitely conclude it if we adhere to antecedently regulated patterns. Now we can fall into dogmatics or, should we say, blindness for logical problems, the way traditional logicians did, who hung onto syllogistic and S-P sentence structures.

The problem presents itself of how to develop this »hierarchy« of logical form, which is provoked by the different contexts with the same sentence, and how to link them to one another so that the »natural« would flow one into another, whereby the deduction systems would similarly change, demanding a certain internal construction of sentences. And, if we turn to Frege once again, as the »paradigm« of contemporary logic, we can say that his functional interpretation of sentences, along with all progress, still contains elements of the »mathematization« of logic, thus that strict »mathematical logic« can exist only in mathematics itself, in the kingdom of algorithms and functions, and that it is a risk to transpose this logic schematically outside this field.

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