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**Mathematical Science of Being**

Mathematics has an important place in the philosophy of Alain Badiou; it is a carrier of a radical, innovative, disciplined, universal, and ontological thinking; it is a scientific truth procedure and ontology. That mathematics can be ontology is not an entirely new idea; it was already addressed by Edmund Husserl in *Formal and Transcendental Logic*, arising from the idea of an all-inclusive science that would be a formal mathematics in its entirety.¹ Husserl’s philosophical position, however, is the transcendental phenomenology of consciousness, or *cogito*, which was objected to by the epistemologies of Jean Cavaillès and Albert Lautman, who argue for the primacy of concept, universality, and non-subjectivity. They perceived mathematics as a rational and experimental science: the former in the dynamic relationship of historical contingency and internal necessity,² the latter in a Platonistic version of understanding the dialectical structures embodied in mathematical theories.³ Under their influence, Badiou took a step further with the thesis that mathematics – in the ZFC version of axiomatic set theory – is ontology. This thesis is in a certain way paradoxical because Badiou’s doctrine of truths is based on the concept of an event, which breaks the ontological laws of being. Thus, mathematics is supposed to be both a science of the ontological laws of being and at the same time a truth procedure that arises from the eventful interruption of these ontological laws.

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¹ Edmund Husserl, *Formal and Transcendental Logic*, trans. D. Cairns, The Hague, Martinus Nijhoff, 1969, p. 77: “Besides set and cardinal number (finite and infinite), combination (in the mathematical sense of the word), relation complex, series, connexion, and whole and part, are such derivatives. Accordingly, it is natural to view this whole mathematics as an ontology (an a priori theory of objects), though a formal one, relating to the pure modes of anything-whatever.”


In the present article, we would like to address this double status of mathematics by following the intimate connection – which Badiou has placed under the wing of the Platonistic orientation – between rational materialism and ontological realism. Under the mark of rational materialism,⁴ we understand the materialist epistemology of true procedures, which Badiou already set up in his youthful writings, although at that time he had not yet developed his doctrine of truths. Rational materialism is a rational materialist epistemology that analyses autonomous, homogeneous, non-empiricist, innovative, and universal materialist truth productions. On the other hand, ontological realism refers to ontology and it is a position in mathematics that confronts mathematical formalism⁵ by pronouncing the firm, real existence of being as being. Taking into account the necessary connection between rational materialism and ontological realism, one can truly understand Badiou’s fundamental proposition that thinking and being are the same.

Badiou believes that there are only four realms of human practices that can generate truths, and they are love, art, science, and politics. Truths are subtracted from general encyclopaedic knowledge and they produce universal and radical truth-thoughts that Badiou names “generic thought.”⁶ Truth-thoughts are generic because they are related to the generic truth procedures that have the ontological status of generic multiples. The generic thought of a concrete truth procedure is a specific thought. It is entirely autonomous and cannot be approached from the outer instance. It means that it cannot be mistaken for either philosophical thought or for knowledge or thought of any other truth procedures. Mathematics as scientific thought is thus the other in relation to philo-

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⁴ We borrowed the expression “rational materialism” from the French epistemologist Gaston Bachelard (Le matérialisme rationnel, 1953). As a synonym for rational materialism, Bachelard uses other terms, such as: applied materialism, scientific materialism, dialectical materialism, technical materialism, materialistic rationality, and materialistic rationalism. In the following, we will also use the term “materialistic rationalism”.

⁵ Badiou rejects the formalistic orientation that understands mathematics as a language game and as a codified formal language producing rigorous manipulation with mathematical concepts and objects, which are merely formal mathematical constructions. For Badiou, Kant’s understanding of mathematics is also a formalism (as a transcendental formalism). Cf. Alain Badiou, Gilles Haéri, In Praise of Mathematics, trans. S. Spitzer, Cambridge and Malden, Polity Press, 2016, p. 30.

sophical, political, artistic, and love thought. Nothing can think (in terms of its production) certain generic and truth-thought but that concrete generic thought itself, because generic thought is strictly bound by the material conditions of its autonomous production protocol. Such thought does not use material and content from external, independent, objective reality, and therefore the material or object of the truth procedure is exclusively produced by the internal process of its truth production.

Generic thought is strictly subjective, but subjectivity here does not relativise the universal and creative value of this thought. The emphasis on the subjectivity of thought means accepting the subjectivity literally as something that cannot possibly be a matter of a correlative object. To be a subject, as Badiou says in his Manifesto for Philosophy, means to be “without a vis-à-vis.” In Anthropology of the Name, Sylvain Lazarus, Badiou's political comrade and a close theoretical colleague, addresses such thought as “thinking in interiority”, and proposes the thesis that thought is a relation of the real and not a relation to the real. Lazarus calls this “an approach ‘in subjectivity’” or thinking “in interiority”. The dialectic of the subject and the object is out of the question because thought is “the subjective without a dialectic.” Or, in the words of Badiou in his analysis of Lazarus: “The whole problem is to think thought as thought and not as object; or again, to think that which is thought in thought, and not ‘that which’ (the object) thought thinks.”

The notions subjective/objective do not have a special meaning here, because what matters is the rational-materialistic logic, according to which: 1) the exterior is disqualified and 2) the internal and homogeneous object, arising from autonomous, generic, truth production, is the only subject of that production. That is why such production is rational and not empiricist. Its subject is not found in the exterior but is rather invented, making this production materialistic, because it brings changes, and intervenes by creating something new, something that had not existed before. In the 1960s, Badiou recognised such production in

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7 Ibid., p. 93.
9 Ibid., p. 3.
10 Ibid., p. 4.
science (particularly in mathematics)\textsuperscript{12} and aesthetics (in Macherey’s concept of literary criticism autonomy).\textsuperscript{13} In the early period of his philosophy, Badiou understood the categories of subject and truth as ideological notions as the opposite of the self-sufficient, autonomous, homogeneous, and \textit{objective} procedures of scientific production that can cut through the tissue of ideology. Under Althusser’s influence, Badiou rejected all humanised versions of the conscious subject as ideological forms of subjectivity, and understood science as a machine of objective mechanisms that break with ideology. Due to the non-intentionality of the mathematical mechanism, mathematics was a strictly non-subjective machine, functioning mechanically and following the pattern provided by the concept of a Turing machine. With the doctrine of truths developed by Badiou at the end of the 1980s, these notions changed. The innovative and universal, arising from the eventful breaking with the existent, became subjective and entered into the service of truth procedures, while the status quo and what is an immediate given gained the status of objective reality. Despite the rotating subjective/objective marks, the rational-materialistic logic remained the same. The swap was thus made without a significant problem, so it is not unusual that Badiou kept the concept of subject as the agent of truth in a paradoxical image of a non-subjective machine, as was lucidly noted by Zachary Luke Fraser, since the new concept of subject is a \textit{matheme}, which defies religious, ethnic, ethical, psychological, conscious, or sociological characterisations.\textsuperscript{14}

This is followed by a question: How does insight into the concept of autonomous rational and materialistic scientific production, posited in the 1960s, help us understand the step that Badiou made in his book \textit{Being and Event} (1980s)? From the concept of mathematics as a scientific mechanical automaton, introduced at the end of the 1960s, in the late 1980s Badiou came to the concept of


mathematics as a scientific truth procedure and ontology, which, historically looking, reached the stage of scientifically pronouncing being as being through the axiomatic set theory of the ZFC system. By taking into account the rational and materialistic character of mathematics as a truth procedure, we will read the equation mathematics = ontology as an abbreviated form of the following description: mathematics = an automated and mechanised machine that uses an axiomatic formalized language as its writing; it is also an experimental scientific practice that produces the object of its own production and operates with its own material that does not arise from the empiricist exterior; and it is also ontology = science that pronounces the form of the multiple of being as being. We must also take into account the paradigm of ontological realism that concerns the ontological status of mathematical concepts and according to which mathematics has an “essential relationship with all there is.” Rational materialism and ontological realism here address each other perfectly well, which is explicitly shown by Badiou’s version of Platonism, which rejects the primary distinction between “internal and external” and between “the known and the knowing mind,” because every assumption upon which a subject aims at an external object (even if the object is ideal) is empiricist. In the mathematical process, there is no subject-object difference, which means that all of the value of the immanent ontological identity, “the same is thinking and Being,” is in that: “In so far as mathematics touches upon Being, it is intrinsically a thought.” This means that Badiou’s thinking of ontology arises out of an implicit idea of mathematics as a materialistic automaton of rational thought requiring special ontological measures, which will be presented below.

Axiomatic Prescription and the Form of the Multiple

In the following, we will focus on the Ideas of the multiple and on the ontological form of multiplicity, giving special attention to the axiomatic prescription of ZFC set theory and its ability to construct compositions based on an empty set. The emphasis on the axiomatic prescription or axiomatic presentation of set theory will lead us to insight into the rational materialistic character of math-
Mathematical production, which corresponds to Badiou's assertion that an axiomatic set is an immanent form of being-qua-being. At the same time, a composition based on an empty set will indicate the moment of being occurring in an axiomatic presentation, which supports the position of ontological realism. In this way, we will be able to show the presentation form of the pure multiple that functions as ontological raw material for the mathematical inscription of being as being. In the materialistic identity of form and matter, we will confirm the intimate connection of mathematical materialistic rationalism and mathematical ontological realism, which eventually means that in mathematics, it is the same to think and to be.

The mathematical theory of the ZFC axiomatic system is a referential place (topos) of thinking being as being, where this “as being” is subtracted from the normative power of the One. The axiomatics and the powerlessness of the One at the level of being are mutually connected, since axioms can be understood as propositions about the multiple that “exclud[e] any explicit definition of the multiple – the sole means of avoiding the existence of the One.” Using an operative form of axiom, mathematical set theory avoids the conceptualisation (a concept is only another way to reinforce the normative of the One) of its material (pure sets). Badiou marks the nine axioms of the ZFC axiomatic system in the Platonistic manner as “Ideas of the multiple,” or, in the Aristotelian manner, as “the ‘first principles of being’.”

There is no object in ZFC set theory, says Badiou in Being and Event, that would be “addressed” by the theory. This non-existence of the object must be understood in a triple manner. The mathematical material is not an object taken from the empiricist experience, which is a directive that already exists in Badiou’s early perception of mathematics as rational materialistic production. This idea is also present in the ontological universe: “It can be described and thought only on the basis of the axioms or principles with which it is consistent. [...] It is rad-

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20 Ibid.
ically unempirical.”21 This is connected to another momentum arising from ontological realism. Mathematics is thought as far as “mathematics touches upon Being.” In this immanent identity of thought and being, however, there is no place for the difference between a knowing subject and a known object. The third momentum is that mathematics as the ontology of non-One does not affirm the form of an object. The ontological form is not a form of the object, which is the form of the One. It is important because of the difference that Badiou articulates between ontology and phenomenology. Within the latter, the form of the object is established as a form of appearance of being.

Given the three characteristics of the non-existence of an object in mathematics, we can further ask what is “it” that mathematics operates with, what is the material that is specific to the mathematical production? If there is no mathematical object (there is no One at the level of being), “what” then “is”?

If there is no object of mathematics, the latter does not present anything, but while it does not present anything (because there is nothing to present), it presents the presentation itself.22 Ontology is a presentation of presentation and thereby a site where we can grasp the general momenta of the structuration of the structure. Ontology is a presentation that does not present an empiricist object; it does not present in the subject-object relation and it certainly does not present in the form of the One. The axiomatic and subtractive character of ontology dictates that we must axiomatise, that we must present the raw matter of ontology (whatever this raw material is) in the form of the non-One. This form of presentation that manages to subtract itself from the normative power of the One is a presentation form of the multiple-without-One.

If ZFC set theory operates with the presentation form of the multiple-without-One, can the conclusion be drawn that being itself is multiple?

We must be careful here. To say that being is multiple is not the same as to say that being is presented in the form of the multiple-without-One. In the latter case, we take into account the ZFC set theory axiomatics, which, within its own

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22 Badiou, Being and Event, p. 7.
23 Ibid., p. 30.
production, presents in the form of the multiple-without-One and thus through mathematical axioms pronounces being as being. Mathematics analyses the structurability of structures within its own mathematical situation, i.e. it analyses what the structures have in common, which is “the fact of being, quite simply.”24 Non-mathematical (non-ontological) or, in the words of Badiou, an un-determined situation “is not such that the thesis ‘the one is not’ can be presented therein.”25 “In an indeterminate situation there is no rebel or subtractive presentation of the pure multiple upon which the empire of the one is exercised.”26 Ontology, as an artificial, highly formalised, experimental, and mathematically defined situation, is the only situation that is capable of the subtractive process. Ontology being mathematics literally means: it is a situation that is ontological, because it is established within the mathematical axiomatic production; ontology is absolutely not transferable, i.e. for a non-mathematical situation (philosophical, artistical, poetic, theological, etc.) it cannot be replaced; only a mathematician is capable of “ontologising” (of ontological activity);27 access to being, contrary to negative theology, is rational and immanent; and finally the prescriptive characteristics of ZFC axiomatic set theory impose the form of presentation that is the presentation form of the multiple.

Ontology, with its subtractive power, prevents being from expressing itself as the being of the One, which, on the other hand, still does not mean that being is multiple: from the fact that there is not the One at the level of being, it does not follow that there is a multiple at the level of being. Being, says Badiou, is entirely heterogeneous to the opposition One-multiple.28 The One is not is a decision-argument that only tells what is not there at the level of being, and accordingly develops the ontological consequence that ontology subtracts or presents, which means that ontology presents by subtracting and thereby produces precisely the presentation form of the multiple-without-One. Between the ontological decision (that there is no One) and being (that we have thus far said nothing positive about; we have only been speaking about it negatively – that there is no One at the level of being), there is an ontological consequence that ontology presents in the presentation form of the multiple-without-One. The multiple is not being,
while at the same time, it has not totally disappeared, since it appears as the
form of presentation: “[T]he one and the multiple do not form a ‘unity of contraries’, since the first is not whilst the second is the very form of any presentation of being.”

The multiple is thus “solely the regime of presentation,” “a figure of presentation,” while the theory of the multiple is a “general form of the presentation of being.”

The question we cannot ignore concerns the role of presentation in ontology, since it is not directly clear what purpose this duplication or mediation with the concept of presentation serves.

Badiou does not always pay attention to the concept of presentation he introduced in Being and Event, where being and multiple are quite often equated, while in Briefings on Existence: A Short Treatise on Transitory Ontology the concept of presentation is almost entirely missing. The consequences of this inexactness of Badiou also emerge in the secondary literature, which tends to put a direct equal sign between being and multiple. We believe that this is a commonly missed point, which may be somewhat understandable, but we must nevertheless insist on the ontological presentation, which is exactly the axiomatic prescription articulated through the form of the multiple. Therefore, we would like to call to mind Badiou’s note to readers that sometimes he himself uses abbreviations which we should not understand literally:

I said that if being is presented as pure multiple (sometimes I shorten this perilously by saying being is multiple), being qua being, strictly speaking, is neither one nor multiple. Ontology, the supposed science of being qua being [...] must present; at best, it must present presentation, which is to say the pure multiple.

Ontology must therefore present presentation, and making an equation between being and multiple is a risky undertaking. However, to understand this point of Badiou’s, we must find out, how Badiou understands the concept of presentation at all.

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29 Ibid., p. 47.
31 Ibid., p. 45.
32 Ibid., pp. 61–62.
There is a remark of Badiou’s in his notes at the end of *Being and Event* that he had borrowed the concept of presentation from Jean-François Lyotard.\(^{33}\) In Lyotard’s book *The Differend: Phrases in Dispute*, there is a chapter entitled “Presentation”, which Badiou refers to in his essay “Custos, Quid Noctis?”, which is his review of *The Differend*. In this text, published before the book *Being and Event*, Badiou also pays attention to the concept of presentation by summarising Lyotard’s understanding of “Descartes’ doubt”. Doubt does not lead to evidence in the form of “I think”, as with Descartes, but to the evidence that what primarily happened was the phrase “I doubt”. Or in the words of Lyotard: “It does not result from the phrase, *I doubt*, that I am, merely that there has been a phrase.”\(^{34}\) This means that “I (speak, think)” is not the evidence with which everything begins, but rather that “I” is a result, the conclusion of the *phrase that occurs before it*.\(^{35}\) Every phrase presupposes other phrases: “It presupposes language, which would be the totality of phrases possible in a language.”\(^{36}\) The multiplicity of phrases is thus something that is *before* the subject or world.\(^{37}\) But this multiplicity cannot be presented: “One can, in fact, describe, *Language is this and that*, but not show. *And this is language*. The totality is not presentable.”\(^{38}\) In Lyotard’s conception of presentation, every following phrase presents a presentation of the previous phrase: “The presentation entailed by a phrase is forgotten by it [...]. Another phrase pulls it back out and presents it, oblivious to the presentation that it itself entails.”\(^{39}\) At the same time, it is true that: “What is not presented is not. The presentation entailed by a phrase is not presented, it is not. Or: Being is not. One could say that when an entailed presentation is presented, it is not an entailed but a situated presentation. Or: Being grasped as an existent is non-Being.”\(^{40}\)

In his notes at the end of *Being and Event*, Badiou says that the presentation, as a pure multiple, belongs to an important theme of the era, which is reflected not


\(^{37}\) Alain Badiou, “Custos, Quid Noctis?”, p. 227.

\(^{38}\) Lyotard, *The Differend: Phrases in Dispute*, p. 59.


\(^{40}\) *Ibid.*
only by Lyotard, but also by Gilles Deleuze. Unlike Deleuze, who thinks this concept under the natural paradigm, and Lyotard, who thinks it in the name of the juridical paradigm, Badiou sets this concept under the wing of mathematics. Nevertheless, certain similarities between Lyotard’s and Badiou’s concepts of presentation still remain. That presentation is a kind of regime of the multiple, which is prior to subject, is also Badiou’s starting point. Lyotard actually provides the idea, 1) that there is something not-subjective, which is “the ultimate existential guarantee,” as Badiou calls it, of the phrase that had happened; 2) what there is, is not presentable in its totality, but merely in its situation (we can also say in its localisation); 3) therefore, what is not presented, simply is not, it is a non-being; 4) and is simultaneously something that is presented with another phrase by being included in the previous phrase. We can find all of these elements later in Badiou. In this way, Badiou also uses a review of Lyotard to announce and promote his own project, which will be a theme of Being and Event. In the same text, Badiou wrote: “I will only say the following – which is close to Albert Lautman’s theses – namely, that mathematics in its history is the science of unpresentable presentation. One day I will prove it.”

The mathematical paradigm, Badiou continues in “Custos, Quid Noctis?”, allows us to think being in a consistent manner exactly like an “existential scission of the nothing and the name,” such as, for example, “the empty (nothing) set (name) exists.” A null-name, which refers to an empty or null-set, will be marked in Being and Event as the proper name of being. Mathematics thinks being in a consistent manner in a form of an existential scission of null and name, which is actually “the logic of scission as [a] form of the occurrence itself.” If we connect these words of Badiou’s with words from Being and Event, we find an interesting momentum. In Being and Event, Badiou says: “the multiple is the regime of presentation; [...] being is what presents (itself).” Being is not a presentation and Badiou requests “the expulsion of any presentifying assumption

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41 Badiou, Being and Event, p. 512.
42 Alain Badiou, “Custos, Quid Noctis”, p. 226.
43 Ibid., p. 237.
44 Ibid.
45 Ibid.
of being”;⁴⁷ what is not presented also is not (in the words of Lyotard), while ontology is a science about being, even if there is no presentation of being (in the words of Badiou). Mathematics is thus “a science of non-presentable presentation” and an empty set is exactly this non-presentable of presentation. Being is thus non-presented, but it also somehow happens to the presentation, it somehow occurs in the presentation; and this event (= occurrence) is reported by ontology, which has the ability to present the presentation itself or the ability to present what “occurs” in the presentation, that is, what happened to the presentation. It is also true that at exactly this point, where being occurs in the presentation, being itself is multiple, “because being is only multiple inasmuch as it occurs in presentation.”⁴⁸ The form in which being occurs/happens/appears in presentation is multiple, because the multiple itself is a regime, figure, and form of presentation.

At this point, we have reached the crucial momentum. The concept of presentation in the regime of the form of the multiple is important, because Badiou has used it to philosophically describe the internal-mathematical activity, which is exactly that the axioms of the ZFC system operate, present, or construct “through” the form of the pure multiple. At the same time, the concept of presentation allows being to occur [French: advient] to the ontological discourse. At the point where being “occurs” to the ontological presentation, the independence of the ontology “object” (being as being) and its “dependence” on the ontological presentation itself (= axiomatic production) are simultaneously confirmed. In this minimal difference between the non-presentable (being) and the occurrence of the non-presentable (being in presentation), being is quilted with an ontological theory of the multiple. This is a point where the simultaneousness of ontologically-materialistic rationalism and ontologically-materialistic realism is actually confirmed. The mathematical axiomatic production of the ZFC system articulates or writes out what has occurred in its axiomatic prescription. The ontological presentation thus does not cause, but still enables, the real of being to occur, without requesting that we understand ontology as a reflection of the being’s objectivity (like some of the vulgar Marxist versions of the mirror theory would suggest). The presentation paradoxically allows being to be present without being presented and thereby lost. Being is included in pres-

⁴⁷ Ibid. p. 72.
entation without being presented; it is present as the non-presentable or as the “‘multiple’ of nothing.” Being is therefore with a negation, i.e. it is subtracted from its own presentation. Being is with a negation, because it has its own presentation subtracted. The One is not is a decision-argument that only says what is not at the level of being, and accordingly develops consequences that have nothing to do with what being as being is. This is important because exactly by saying nothing about being, the decision says something important about being: being is bound to nothing, which is the nothing of the subtracted presentation. Being as being “strictly speaking, is neither one nor multiple.” When we do not confirm anything positive about being, we confirm being itself, which is the null-presentation.

At this point, we need to be careful not to attribute to the negation more than what the ontology’s intrinsicality allows. We must not understand the primary negation as an example of primality of logical laws over ontological laws, since it is a consequence, not a principle. In the mathematical ontology of the ZFC system, we can recognise the rules of classical logic, but it is not because ontology would adhere to these rules a priori, but because being itself in mathematics is articulated and expressed classically. Mathematics sets the axiom of the empty set, which literally decides the existence or the primitive name of being. The absolute initial point of being is articulated with the empty set axiom, which states that there is a set without elements. Being and ontological prescription (through mathematical axioms) are thus in a mutual grip. The ontological prescription is inevitably immanent to the mathematical procedure, thereby making axiomatics “the mathematical realization of the proper formalization of this decision.” If we can talk about the a priori conditions of ontology at all, then such a condition would be that the intrinsic discourse about being as being must be developed in the frame of a situation that is necessarily ontological (= mathematical), which is only another way of saying that mathematical access to being requires that one understand ontology as an autonomous rational materialistic production.

49 Ibid., p. 71.
50 Ibid., p. 61–62.
52 Ibid., p. 3.
This entails a new characteristic of ontology; namely, it is an absolute referential universe, which is “absolutely intelligible on the basis of nothing.” Badiou says: “In Set Theory, the primitive name of Being is the void, the empty set. The whole hierarchy takes root in it. In a certain sense, it alone ‘is’.” The absolute initial point of being is expressed by the axiom of the void-set, which states: “There exists a set which has no element.” An empty set is a universal constructor of different structures that form rational orderliness and fascinating mathematical architecture, where “special rules” are involved and dictated by the axioms of the ZFC system. Ontology is a militant, rebellious situation exactly because it is axiomatically formalised, which enables it to operate with only one set type, namely with the pure set (= the pure multiple). Pure sets are pure exactly because they contain nothing else than sets, which contain nothing else than sets, and so on until the empty set, which is the first case of a pure set, which everything begins with. “In other words, a pure set is either the empty set or a set that contains other pure sets.” What is repeating, here, is not only an empty set, but also the very form of the set, the form of the multiple of multiples, which is “the form of presentation itself,” indicating that the structurability of these structures is presented in the form of the multiple. The axiomatical presentation regime of the ZFC system is a regime that dictates the form of the multiple. We must not forget that a pure set, the mathematical set that ZFC theory operates with, is not defined or conceptualised and it primarily denotes a certain form and not a concept of mathematical axiomatic operativity. A pure set is simply an axiomatic prescription, a presentation. Axioms are applied to the variables α, β, γ, etc., “in respect of which it is implicitly agreed that they denote pure multiples.” Here, we are dealing with formal inscriptions, where α is of the same “scripture type” as β, which confirms that within the ontological situation, a uniformity of pure multiples exclusively prevails. Ontology encounters nothing that would be external to it or that would be heterogeneous to the pure multiple. By the example of the axioms of the powerset and of union, B-
Badiou stresses that whether we go to the composition or internal decomposition (dissemination) of a given multiple, “the theory does not encounter any ‘thing’ which is heterogeneous to the pure multiple.”\(^{60}\) The axiom of replacement (or of substitution) is particularly interesting in these terms, since it expresses exactly that, even if the elements of a given set are replaced by other elements, the result of this replacement is still a set. The implementation of an axiom is different to the “content” of the multiple, because “the attribute ‘to-be-a-multiple’ transcends the particular multiples which are elements of a given multiple.”\(^{61}\) The main point is that despite the replacement of the elements, the form of the multiple remains. Given that nothing can obscure the uniformity of presentation in the form of the multiple, we can confirm that the homogeneity principle is an essential presentation principle of being in the theory of the multiple.

When a concept fails, we do not enter chaos, but a form. The form is a rebellion against the concept and is essentially a form-without-concept (reminiscent of Kant’s concept of the two forms of intuition, space and time, which, strictly speaking, are not concepts). The presentation form of the multiple-without-concept (understood in the instance of the mathematical letter) is the ontological form of mathematical operativity as ontology. Ontology operates with the ontological form and it simultaneously also operates on it, because the set, “the Multiple, for mathematics, was not a (formal) concept, transparent and constructed, but a real whose internal gap, and impasse, were deployed by the theory.”\(^{62}\) Badiou also says the following: “I began to think that if mathematics achieves the secrets of thought it was because of the type of thinking that it is. My conception of ontology began to follow this line of thought as well as the idea that the most sedimental thing will be pure multiplicity.”\(^{63}\)

The pure multiple is not merely a form of presentation, but also a moment of the real, which is “the most sedimental thing” without a concept. This thing-without-concept that ZFC set theory operates on is the most primitive ontological raw material of mathematical production. The pure multiple is the matter that is at

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\(^{60}\) Ibid., p. 69.

\(^{61}\) Ibid.

\(^{62}\) Ibid., p. 5.

\(^{63}\) Alain Badiou, Zachary Luck Fraser, and Tzuchien Tho, “The Concept of Model, Forty Years Later: An Interview with Alain Badiou”, in A. Badiou, *The Concept of Model*, pp. 79–106, 103.
the same time the ontological form of the multiple. This equation of matter and form is possible in mathematics because ZFC set theory is an inscription of being in the instance of a letter. The formal inscriptions $\alpha$, $\beta$, $\gamma$, etc., are not only formal, abstract marks of the multiple. They are much more. The mathematical letter tells us that the multiple is and it lacks a definition of what this multiple is, it is the “inscription without concept of that-which-is.”\footnote{Badiou, \textit{Being and Event}, p. 48.} Mathematics “is the \textit{literal inscription} of being,” and not a description of being.\footnote{Jon Roffe, “Alain Badiou’s Being and Event”, \textit{Cosmos and History: The Journal of Natural and Social Philosophy}, 2 (1&2/2006), pp. 327–338, 330.} It is a discourse that “refers to nothing other than itself” and its marks or letters are “the sole reality of mathematical discourse.”\footnote{Ibid.} Being as being is thus not articulated through the abundance of some ontological Presence, but rather in a formalised letter;\footnote{For the concept of letter as a primary condition, \textit{cf.} Justin Clemens, “Letters as the Condition of Conditions for Alain Badiou”, \textit{Communication & Cognition}, 36 (1&2/2003), pp. 73–102.} ontology “marks the absence of being qua being in through the agency of the letter.”\footnote{Roffe, “Alain Badiou’s Being and Event”, p. 331.}

This of course does not mean that Badiou advocates mathematical structuralism,\footnote{\textit{Cf.} Baki, \textit{Badiou’s Being and Event and the Mathematics of Set Theory}, p. 88.} because even if every mathematical proof is “bound to a letter, it cannot be reduced to it.”\footnote{Alain Badiou, ‘Préface de la nouvelle édition’, in \textit{Le Concept de modèle}, Paris, Fayard, 2007, pp. 27–28; taken from Fraser, “The Category of Formalization: From Epistemological Break to Truth Procedure”, p. xxxvi.} The form of the multiple-without-One or a pure set reveals the inexhaustive richness and “the banality of manifold-being.”\footnote{Badiou, \textit{Briefings on Existence}, p. 30.} The empty set allows us to think consistently very inconsistent and paradoxical “entities”, like asserting there is a set of all triangles with four sides, thereby confirming the existence of inconsistency as such. Being as being is inexhaustive, because the inconsistency is inexhaustive. What is presented of being as being is, on the other hand, consistent and structured in the form of the presented multiple. So, there is \textit{being as being, the inexhaustive structurability of the structure} or the \textit{non-presented presentation of the presented}, which is the \textit{inconsistency} of consistency, occurring as an \textit{empty set}, which is included in every pure set; it is not a \textit{structure (the presented multiple, or consistency)}, even though the \textit{structurability}
(presentation, inconsistency, or an empty set) is what we operate on, what is the matter itself of any structure of a presented, consistent multiple.

The ZFC set theory thus provides an ontological aspect of any given structure, but it does not mean that an empirical or non-mathematical situation arises from the mathematical situation or is based thereon. Badiou is not a philosopher of “the ontological levels of reality” (like Nicolai Hartmann’s ontology), neither does he advocate a discursive hierarchy (like Rudolf Carnap does). For Badiou, a mathematical set is simply a “modus of thinking”, which thinks ontologically and is capable of thinking the firm point of the real. This real is not a matter of concept, but a matter of form, which is the matter itself. At the level of being and its presentation within ZFC axiomatic theory, it is the same to be, to think, and to formalise or form-realise. We can conclude that ontological rationalism and ontological realism in Badiou’s version of the intrinsic and Platonistic ontology are not mutually opposing, but rather complementary.

The Ontological Decision and Metaontology

Let us conclude with the elementary question: With what kind of concept of ontology does Badiou operate? In an Aristotelian manner, Badiou defines ontology as discourse on being as being, which is an intrinsic understanding of ontology. Jean-Toussaint Desanti stresses that intrinsic ontology can be defined according to the minimum or maximum domain of interpretation based on how one thinks being as being (on é on). The maximum interpretation will exhaustively develop all the conceptual richness of being as such, while the minimum interpretation will be limited to only the essential, asking: “[W]hat is the least that must be thought in order to define the status of the proposition ‘there are beings’?” Badiou opts for the minimum interpretation of intrinsic ontology, which ena-

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75 Ibid.
bles him to divide the history of ontology in relation to the minimum question: Should we think being-as-being as being-of-One or as being-of-multiple?

To this minimum question, however, philosophy does not offer a minimum answer. Arguments come from both sides: the Parmenides-Zeno line paved the way for ontology under the sign of the One, while the Democritus-Lucretius line did so for the ontology of the multiple. Both lines, however, started a history of a dramatic ontological struggle that has left us with only a portico of the ontology of its “ruined temple.” The ruined temple of ontology is nothing but a metaphor for constructing something that is destroyed in the process of construction. In Being and Event, Badiou compares philosophy with the phoenix, the mystical bird dying and rising renewed from its ashes. Caught in a vicious circle of self-destruction and self-awakening, philosophy is a “phoenix of its own sophistic consummation.” The destructive cycle can be interrupted by admitting that the rational knowledge of philosophical argumentation is clearly not the one that leads to a final solution, since the philosophical argument itself destroys the possibility of a final ontological proof. These problems came to philosophers’ mind quite early. In Parmenides, Plato tried to systematically derive argumentations from both sides and thoroughly examined, in the form of a logical exercise, all of the possibilities implied by the assumption that the One exists, as well as the assumption that the One does not exist. And in the last line of Parmenides, he captures the very essence of the impossibility of the philosophical derivation (argumentation): “if ‘one is’ or if ‘[one] is not’, [then] it and the others both are and are not, and both appear and do not appear to be all things in all ways, both in relation to themselves and in relation to each other.”

Badiou intervenes exactly at this point. If we dismiss Plato’s conclusion as nonsense that has brought us to the end of argumentation, we admit the weakness and collapse of thought. The other option is to see only the beginning in this impossibility, meaning that we recognise “the first example, though a purely philosophical one, of an absolutely undecidable argument.” The latter option defends the rational argumentation that does not lead to the final ontological

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76 Badiou, Being and Event, p. 25.
77 Ibid.
79 Badiou, Briefings on Existence, p. 92.
proof, but to the final ontological decision in favour of the existence of the One or multiple at the level of being. Badiou's point is that at the level of being we cannot make a rational deduction of the ontological proof, because that leads to the absurd; instead, we can decide that the One does not exist at the level of being. This decision allows Badiou to claim that ontology is mathematics, because it is precisely the mathematical set theory of the ZFC axiomatic system that separates being from the One and consistently pronounces the consequences of the decision that there is no One at the level of being.

We could ask: Why is it so?, Why is there not One at the level of being, i.e., what were the arguments that initially convinced Badiou to put his philosophy under the condition of the ontology of the not-One?

However, we must warn that Badiou is not obliged to answer our question. At the beginning of Being and Event, Badiou accepts the thesis that “the one is not,”80 without offering an explanation of this thesis, either ontological or philosophical. It is true that Badiou presents the consequences of the ontology of the One in several places, but this presentation is only an address on behalf of the opposite position. In the ontology of the One, he sees the weight of the religious, metaphysical, and poetic discourses that maintain the teleological and theological approach to being, and are the ontological backside of phenomenology, which insists on the humanistic motive of finitude. His intention is to avoid this motive, which led him to condition his philosophy with mathematics; however, the purpose thereof is not a rational justification of the initial decision that mathematics is ontology, which confirms that there is no One. The rational justification of the ontological decision operates as a rational argumentation that must be confirmed based on itself and thus cannot be bound to ethics or politics, regardless of the emancipatory motives that may be there. Confirmation that refers to itself is not a definition of its contents. A decision is exercised without defined elements, meaning that it is “proven” without an explicit proof. So, what is this proving-without-proving?

As Burhanuddin Baki says, Badiou implicitly refers to both theses – that mathematics is ontology and there is no One – to a consistency proof, which differs

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from a *direct proof*. A *consistency proof* is used in relation to undecidability, while a *direct proof* is “the mathematical condition for what we usually understand as logical argument.”

Badiou’s genius – which must be acknowledged – was to recognize the notion of consistency proof as another reasonable mathematical figure from which an alternative form of argumentation can be conditioned. We do not need to argue the conclusion from our basic assumptions; we can just demonstrate that it is not illogical for us to decide the conclusion to be true. Between the axioms and the end result lies a decision, a militant commitment. This gives us a great flexibility for pursuing any line of reasoning. Whenever a detailed and straightforward argument cannot be given, one can decide for the proposition to be true, with this deciding not being a simple recourse to subjective prejudices, but to the event of decision itself, which Badiou will later link to an event of subjectivity itself, the emergence of a new subject.

In *Being and Event*, Burhanuddin Baki stresses, we will not find direct argumentation of the theses that there is no One and that mathematics is ontology. “Badiou only claims that there is nothing wrong with taking Being to be multiple and mathematics to be equivalent to ontology. The validation of the propositions *does not precede* but *comes after* the decision.” Exactly in this militant commitment lies the “fidelity [fidélité] to the decision,” whereby the consistency proof is “a license for decidability.” Badiou finds this license for decidability in the revolutionary technique of “forcing”, which was introduced in mathematics by Paul Cohen. It is Cohen’s method of proving or forcing the consistency, which is strictly different than the causality procedure. Badiou recognises its value particularly when it comes to the truth procedures that, as productions of something new, cannot be simply deduced from an existing situation. Even though truths break with a given situation in such a way so as to expose the inconsistен-

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81 Baki, *Badiou’s Being and Event and the Mathematics of Set Theory*, p. 84.
82 *Ibid.*, “In the case of direct proof, there is a linear and necessary deductive thread from the premise to the propositional end result. But in the case of consistency proofs, the proposition is only decided to be true, and that is already enough.” *Ibid.*, p. 85.
83 *Ibid.*.
85 *Ibid.*.
cy of the situation, they are not inconsistent themselves, but rather subjectively decided towards productions (forcing) a new consistency. Thus consistency and decision are related. Or, to express that by using the logic of consequences that Badiou introduces in his book Logics of Worlds: “A truth affirms the infinite right of its consequences, with no regard for what opposes them.” This means that Badiou recognised the events (the Cantor-event, the Gödel-event, and the Cohen-event) in the historical development of mathematics that led him to the recognition that mathematics is actually a truth procedure (it produces the new and universal from the eventful interruption), which can – exactly because it is a truth procedure – produce rational consequences (from the Cohen-event) that justify the ontological decision that mathematics is ontology, which articulates being without the One (from the Cantor-event).

Truth procedures are concerned with the production and construction of such a world where the decided theses are true. With his book Being and Event, Badiou thus hopes to bring attention to the possibility of the existence of a world (model) where both theses (mathematics as ontology and the non-being of the One) are true. Badiou cannot prove this by forcing out this truth in the scope of philosophy, because philosophy is not a truth procedure. Therefore, Being and Event is not a direct spot to look for forced proofs that ontology is mathematics and that there is no One; these proofs are in a way a matter of the historical development of mathematics itself. Even if we spontaneously take both theses – mathematics is ontology and there is no One – as consequences of the philosophical decision (as a decision of the philosopher Badiou), it still needs to be stressed that the philosophical decision is not the subject of an ontological decision, but, on the contrary, its consequence. Mathematics cannot adhere to philosophical principles, conditions, or decisions because they are external to it. At the same time, there is no personalised or transcendent decision-maker to decide upon being as being in mathematics as a not-intentional, yet subjective, autonomous, rational materialistic production. All we can say is that there are different operative mechanisms in mathematics that decide like mathematical axioms because

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87 Ibid.
“the axiom is a regime of decision.”90 The task of philosophy is then to position itself under the condition of what was decided in mathematics.

That philosophy is conditioned by mathematics, however, enables philosophy a vast range of philosophical approaches, since despite ontology being mathematics, *mathematics does not know anything about its ontological status*. Philosophy is the one to declare the thesis that ontology is mathematics. That mathematics is ontology, Badiou says, is a meta-ontological or philosophical thesis,91 which is not a philosophical directive for mathematics, but an insight that philosophy gains by its conditioning with the mathematical truth procedure. In this conditioning, philosophy is closest to mathematical thought, because the “conditioned concept of philosophy [...] is a way of philosophy trying to articulate the real from which it accepts its condition.”92 When philosophy thinks mathematics as a thought, it simultaneously thinks mathematical thought as a thought that touches being: “In so far as mathematics touches upon Being, it is intrinsically a thought. Reciprocally, if mathematics is a thought, it touches upon Being itself.”93 In the spirit of Badiou’s early epistemology, we can say this as follows: mathematics is a thought that works like a machine, without self-reflection about its thought and its object, but precisely as “machine thinking”, mathematics is a thought that thinks being. Philosophy is not ontology, but when it thinks mathematics as a thought that as a thought thinks being, philosophy is meta-ontology. This means that by recognising that mathematics is a thought, philosophy accepts that being can be thought (in this respect, philosophy differs from sophism).

Despite philosophy being external to mathematics, mathematics is not external to philosophy, but is the philosophical condition that philosophy opens in its meta-ontological momentum. We, therefore, do not agree with Bruno Bosteels, who marked Badiou’s philosophical undertaking in *Being and Event* also as an intermediation between intuitive approximation and strict mathematical for-

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90 Alain Badiou, Zachary Luck Fraser, and Tzuchien Tho, “The Concept of Model, Forty Years Later: An Interview with Alain Badiou”, p. 101.
malisation. Philosophy does not have the role of a mediator; it is autonomous. The conditionality of philosophy is not its subordinance. Philosophy does not translate mathematical formulae for non-mathematical readers (there are much more efficient manuals for this purpose than philosophical books), but it positions itself under a condition that provides philosophy with the insight that mathematics is ontology. Philosophy takes care of its own conditionality, because otherwise there is no instance that would tell philosophy that mathematics is ontology (mathematics is the last one to know anything about it). Mathematics is in a privileged position vis-à-vis being, which nevertheless has its limits. Although being as being is articulated in mathematics, it is articulated at the point of mathematical ignorance and oblivion. In “Appendix 2” of Being and Event, Badiou says that: “It is necessary to mathematics to forget being in order to pursue its pronunciation,” because being as being itself “does not want to be written.” The highly demanding technical needs of mathematical science force mathematics to forget the ontological destiny of its discipline. Similar applies to the being of truth. Badiou differentiates between the de facto and de jure existence of truths and uses this difference to open a space to position philosophy between the structural ignorance of mathematics and its truth production. For Badiou, a generic set is mathematical de jure “proof” that the being of truths exists. However, mathematicians do not know that and consequently do not know “how to name what was happening there.” Mathematics is a truth procedure that does not recognise its de facto truth status; it is philosophy that can proclaim the de facto existence of truths, due to its status of being conditioned by truths. The limitation that prevents ontology from speaking for itself puts philosophy in a special position: philosophy as the meta-ontology that can speak in the name of ontology, but again, not as a meta-language, but by “adopting” the voice of ontology. Only in one instance of Being and Event does Badiou “allow” ontology to speak for itself, namely in meditations 33 and 34. These are the meditations summarising Cohen’s theory. Badiou says: “I have deliberately weakened the explicit links between the present conceptual development and the mathematical doctrine of generic multiplicities in order to let ontology ‘speak’,

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95 Badiou, Being and Event, p. 468.
96 Ibid.
97 Ibid., p. 360.
eloquently, for itself.” Other than this exception, everything in *Being and Event* is a philosophical prosopopoeia: mathematics as ontology has lent its voice to philosophy, i.e., philosophy has borrowed its voice from ontology. With its specialised formalisation, mathematics articulates being as being, without knowing it, which opens the room for philosophy that is conditioned by mathematics at the point of this ignorance.

The science of being as being has already existed since Greek times and “the sense and status of mathematics” is to pronounce it through its historical becoming. The story of being existed, even if we never had the means to unriddle it until now, which means that being is independent of the instance of the comprehending (transcendental or any other) subject. The question is thus not “How is pure mathematics possible?” and the answer to this question is not “thanks to a transcendental subject.” In these terms, we must “water down” the Kantian concepts and develop a critique of every critique. A critique of every critique is articulated by Badiou with reference to Mao Tse-tung: “We will come to know everything that we did not know before.” Before mathematical set theory emerged, we had not had the means to determine that it is mathematics with its production that “lets being speak”. But now, after all the pioneering and militant “mathematico-logical revolution of Frege-Cantor,” we have realised what we had not known before. Now, it is the task of the philosopher to decide whether to accept the truth of mathematics that mathematics is ontology, which is nothing more than accepting the scientific truth of being that it is not the being of the One and it is the same to think and be.

**References**


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102 *Ibid*.
103 Badiou, *Being and Event*, p. 2.


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