MODELLING FLOW OF SUBTERRANEAN PIVKA RIVER IN POSTOJNSKA JAMA, SLOVENIA
MODELIRANJE TOKA PODZEMELJSKE PIVKE V POSTOJNSKI JAMI, SLOVENIJA

Georg KAUFMANN1, Franci GABROVŠEK2 & Janez TURK3

Abstract

Georg Kaufmann, Franci Gabrovšek, Janez Turk: Modelling cave flow hydraulics in Postojnska jama, Slovenia

The sub-surface flow path through the Postojnska jama cave system has been monitored with 7 stations distributed along the flow path, monitoring stage and temperature. We have used the stage data to model flow through the cave system with the program package SWMM, simulating the active parts of Postojnska jama with simplified geometry. From the comparison of stage observations and predictions, we identified key sections in the cave, which control the sub-surface flow, such as passage constrictions, sumps and by-passes. Using a formal inverse procedure, we determined the geometry of this key sections by fitting predicted to observed stages, and we achieved a very high degree of correlation.

Key words: modelling, hydrology, Postojnska jama.

INTRODUCTION

Some of the world’s prominent cave systems are active water caves fed by sinking allogenic rivers (Palmer 2007; Ford & Williams 2007). Among them are some of the most important caves in Slovene Classical karst such as Škocjanske jame, Postojnska jama and Predjama, where the allogenic streams originate from the adjacent flysch basins (Gams 2004). The sinking streams of these systems are characterised by high flow variability. An extreme example is the Reka river, which sinks into Škocjanske jame; in extreme draught the river may not even reach the cave, while the highest recharge surpasses 350 m³/s (Gabrovšek & Peric 2006). Although the speleogenesis of the aforementioned caves is driven by the sinking streams, the geometry of most of the cave systems is not equilibrated to the high floods: The area of Classical karst is tectonically very active and systems are subjected to continuous changes of boundary and structural conditions, which drive them out of equilib-

1 Institute of Geological Sciences, Geophysics Section, Freie Universität Berlin, Malteserstr. 74-100, Haus D, 12249 Berlin, Germany, e-mail: georg.kaufmann@fu-berlin.de
2 Karst Research Institute ZRC SAZU, Titov trg 2, 6230 Postojna, Slovenia, e-mail: gabrovsek@zrc-sazu.si
3 Janez Turk, Slovenian National Building and Civil Engineering Institute (ZAG), Dimičeva ulica 12, 1000 Ljubljana, Slovenia, e-mail: janez.turk@zag.si
Received/Prejeto: 29.09.2015
High recharge variability is manifested in active caves by fast and high fluctuations of water levels. In case of Reka river, the water in some caves rises for more than 100 m (Gabrovšek & Peric 2006; Cucchi & Zini 2002).

Continuous monitoring of recharge and other physical parameters of ground water flow in caves (e.g. level, temperature) is nowadays readily available and cheap by the use of autonomous loggers. Interpretation of such records may give new insights into the structure of conduit systems and enable us to infer the geometry of parts of the cave systems that have not yet been physically explored (Gabrovšek & Turk 2010). Such interpretation can be based on different types of models; however, if the data are interpreted in terms of the geometry of conduit system, a distributed hydraulic model is the most appropriate choice. Such model must simulate time-dependent flow in epiphreatic channels, where flow conditions change from open channel to pressurised and vice versa. Recently, the EPA Storm Water Management Model (SWMM) has been used to model variable flow through the system of mature karst conduits by several authors (e.g. Campbell & Sullivan 2002; Peterson & Wicks 2006; Gabrovšek & Peric 2006; Turk 2010; Chen & Goldscheider 2014).

Several investigations of flow through cave systems have been based on modelling with SWMM. Campbell and Sullivan (2002) simulated flow through Stephen’s Gap Cave (Alabama, USA) to evaluate the temporal behaviour of the hydraulic head in the cave resulting from the time-dependent inflow from a surface stream. They concluded that both rising of water level during flood conditions and falling of water level during flood recession follow the same pattern, thus no hysteresis occurred in the stage-discharge diagram. Peterson and Wicks (2006) applied SWMM to the Devil’s Icebox-Connor’s Cave System (Central Missouri, USA). They compared predicted discharge with observed discharge and varied model parameters such as conduit geometry (height and width of conduits) and flow properties (roughness of conduits). They found a strong dependence of the predicted discharge of the Manning roughness coefficient and the conduit geometry, but less dependence of conduit slope. Wu et al. (2008) simulated the spring response of Shuifang karst spring (Jinho Mtn., Chongqing, China), which drains a mature karst aquifer via a well-developed conduit system. They concluded that the predicted spring response is well characterised by a conduit system as modelled by SWMM. Chen & Goldscheider (2014) used the SWMM model and a genetic algorithm to simulate flow in the alpine karst system of the Hochifen-Gottesacker in the European Alps and were able to predict both low- and high-flow conditions in the system.

Alternative model techniques for flow and transport through cave systems often follow a MODFLOW-and MODFLOW-CFP-based approach (e.g. Dogwiler & Wicks 2004; Shoemaker et al. 2008; Mayaud et al. 2014; Reimann et al. 2014).

In this work, we apply SWMM (Rossmann 2010) to discuss the dynamics of water levels in the active channels of Postojnska jama in terms of conduit geometry. Turk (2010) in his PhD thesis demonstrated a good performance of SWMM to model stage data in Pivka channel. We expand this model with a formal inverse procedure, determine channel geometries of important flow constrictions in Postojnska jama, and discuss the downstream contribution of the model towards Planinska jama as the major karst spring. The original contribution of this work is application of inverse techniques to identify and estimate the crucial geometrical parameters which govern most of the dynamics.

**STUDY AREA**

Postojnska jama (Postojna Cave) is the second longest cave in Slovenia. The explored system is currently (August 2015) over 24 km long with five known entrances. The system extends within the Upper Cretaceous bedded limestone and hydrologically connects the Pivka basin with its flysch cover to Planinsko polje (Fig. 1), the most NW positioned among all active Dinaric poljes (Note that Logaško polje is no longer an active polje with flat bottom).

The broader region belongs to the thrust belts of external Dinarids (Fig. 1), which form a rim of high topography above the Istria peninsula, the remaining outcrop of the Adria microplate. The Upper Cretaceous limestone belongs to the Adriatic-Dinaric carbonate platform, whilst the Eocene flysches presents the product of the platform’s disintegration.

The area of the cave is located between two major Dinaric faults, Predjama fault (PRF) and Idrija fault (IDF, Fig. 1). Čar and Šebela (1998) distinguished several generations of thrusting and folding in the area since the deposition of Eocene flysche. Cave passages are formed along flanks of Postojna anticline and generally follow
strike and dip of bedding planes. The system extends along two principal levels, the upper level comprised of older inactive passages which is used for tourism, and a lower level with the active stream of Pivka river.

Pivka river is the main allogenic input from the adjacent flysch basin ($Q_{\text{min}} < 0.01 \text{ m}^3/\text{s}$, $Q_{\text{max}} > 60 \text{ m}^3/\text{s}$, $Q_{\text{av}} = 5 \text{ m}^3/\text{s}$) (Frantar 2008). The active flow can be followed without diving for most of the initial 3.5 km, then the flow continues through a series of sumps connected by open surface river channels towards Planinska jama (Planina cave), a large spring cave at the rim of Planinsko polje (Planina polje).

The evolution of cave system has been controlled by active tectonics (Šebela 1998, 2010) and interrupted by related breakdown processes. It passes several fractured/crushed zones, where the breakdown chambers have formed. Allogenic cave sediment have also played important role in the cave development. Many of the galleries have been partially or fully filled by sediments with paragenetic features are observed at many places.

Fig. 1: Top: Topographical map of Western Slovenia. Major faults are mapped as solid and dashed black lines (based on Placer (2008)): Periadriatic Fault (PAF), Sava Fault (SAF), Idrija Fault (IDF), Predjama Fault (PRF), Želmljse Fault (ZEF), Borovnica Fault (BRF). Major cities are marked as red dots, country borders as white lines. Bottom: Topographical map of the Notranjski Kras region. Major faults are mapped as solid and dashed black lines, and rivers as blue lines. Red dots are sinks, blue dots are springs, white dots are dolines. Mapped cave systems are indicated as red lines (PO-Postojnska jama, PL-Planinska jama, T-Tkalca jama, Z-Zelške jame, K-Karlovice).
SAINT-VENANT EQUATIONS

Mature active cave systems such as the systems in the Notranjski Kras have much in common with man-made drainage and sewer systems, as both can be characterised by fast flow through large pipes and channels, with quick reaction to changes in inflow. The Storm Water Management Model (SWMM) from the Environmental Protection Agency (EPA) of the United States is a tool to assess flow and transport through networks of channels, pipes and man-holes, which can be driven by complicated inflow from various sub-catchments (e.g. Rossman 2010). Modelling of water flow in the inter-connected network is based on the solution of the full Saint-Venant equations for time-dependent flow.

The Saint-Venant equations are derived from the Navier-Stokes equation,

\[ \rho \frac{\partial v_i}{\partial t} + \rho (v_j \frac{\partial}{\partial x_j} v_i) = - \frac{\partial p}{\partial x_i} + \eta \frac{\partial^2 v_i}{\partial x_j^2} - \rho g \delta_{i2}, \tag{1} \]

and the continuity equation,

\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_i)}{\partial x_i} = 0. \tag{2} \]

Here, \( \rho \) [kg/m\(^3\)] is density, \( v_i \) [m/s] is velocity, \( p \) [Pa] is pressure, \( g \) [m/s\(^2\)] is gravitational acceleration, \( x_i \) [m] are the coordinate directions, \( t \) [s] is time, and \( \delta_{ij} \) is the Kronecker delta. The first term of the left hand-side of (1) describes the resistivity of motion through inertial forces, the second term particle movement through advection, the first and second term of the right-hand-side particle movement initiated by surface forces, and the third term movements as a result of gravity.

We now restrict the Navier-Stokes equation to one dimension, with the \( x \)-axis along the channel direction, and we drop the viscous term in favor of formulating viscous forces as volume forces, thus adding it to the gravity term. We then can reformulate (1) to

\[ \rho \frac{\partial v_x}{\partial t} + \rho v_x \frac{\partial v_x}{\partial x} = - \frac{\partial p}{\partial x} + f. \tag{3} \]

Here, \( f = f_g + f_v \) represents the sum of gravity (\( f_g \)) and viscous (\( f_v \)) volume forces. We need to define the gravity component along the channel direction, and therefore define a slope \( a \) of the channel, thus the gravity term reads \( f_g = \rho g \sin a \). If the angle is small, we can approximate \( \sin a = a = s \) with \( s \) [-] the slope of the channel. We argue that the viscous force resisting flow is also proportional to slope, but use a different slope \( s_f \) [-]. The sum of the gravity and viscous terms then reads \( f = \rho g (s_f - s) \).

Inserting (4) into (3) and assuming a hydrostatic pressure distribution, \( p = \rho g h \) with \( h \) [m] the hydraulic head, we arrive at the Saint-Venant equations:

\[ \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + g \frac{\partial h}{\partial x} + g(s_f - s) = 0, \tag{5a} \]
\[ \frac{\partial A}{\partial t} + \frac{\partial Av_x}{\partial x} = 0. \tag{5b} \]

The second equation results from integrating the continuity equation in (2) over the cross-sectional area \( A \) [m\(^2\)] of the channel. The Saint-Venant equations are two coupled partial differential equations, which need to be solved for the two unknowns velocity \( v_x \) and hydraulic head \( h \), subject to appropriate boundary conditions.

The friction slope \( s_f \) is based on the empirical Manning equation:

\[ s_f = \frac{n^2 Q^2}{A^2 R^4 / 3}, \tag{6} \]

with \( n \) [-] the Manning roughness coefficient, \( Q \) [m\(^3\)/s] the flow rate, and \( R \) the hydraulic radius.

MODEL

We provide a model of the active parts of the Postojnska jama system, as here we can compare stage model results to stage observations from different parts of the cave system (Gabrovšek et al. 2010). The model, however, extends towards the Planinska jama, the outlet cave of the water passing through Postojnska jama. The unknown part between Postojnska jama and Planinska jama is estimated.

BASIC GEOMETRICAL FEATURES OF THE PIVKA CHANNEL

A map of the Postojnska jama system is shown in Fig. 2. The cave passages follow two main directions, the Southeast-Northwest Dinaric direction, and a system of NE-SW trending faults. The sub-surface flow of the Pivka can be traced through the sample sites (white
diamonds) from the Pivka Ponor to Postojnska jama in the south (1) towards the terminal sump in Pivka jama (beyond 7) in the north-east. This main active system generally represents large channels with cross sections from 2–3 m by 10 m close to the Pivka Ponor up to 10 by 10 m in the southern Pivka jama. Flow is mainly guided through epi-phreatic passages, which are large enough to accommodate even large floods. However, several constrictions occur along the flow path, and these constrictions control the flow behaviour of the entire system, as we will show later.

While passages between sample points 1 and 2 are generally large, a first constriction appears between the Lower Tartarus (2) and Otoška jama (3), with passage width going down to 1.5 m. After Otoška jama (3), passages are wider again, but flow becomes obstructed by Martel’s rockfall (4), a major collapse zone blocking the entire passage. Water flows here through small bypasses in the rockfall, thus this zone is prone to flooding during higher flow conditions. Several flow paths exist, and the rockfall can be traversed through a small passages several meters above the low-flow condition. The Martel Chamber (5) is another obstacle to flow, with several passages active during different flow conditions. Base flow is guided through a small phreatic passage not yet explored. Excess water above base-flow conditions drains through explored passages in about 3 and 10 m elevation above the active river. Upstream from Martel Chamber, some of the water is redirected through an overflow side passage towards the east at high flow rates. This water joins the main passage further downstream.

After the entrance to Magdalena jama (6), the passage is sizeable, but sumps twice, with a phreatic loop going down to −17 m. At higher flow rates, some of the water is redirected through an overflow side passage towards the east and joins the main passage further downstream. The last part of the active cave, Pivka jama (7), is characterised by large rather steep passage (> 10 m x 10 m) which leads to the terminal sump of Pivka jama. This sump is about 30 m deep and 150 m long. Behind the cave continues with river galleries of large dimensions (> 10 m x 10 m), interrupted by three more sumps northwards. Then the channel turns SE along the Dinaric direction and sumps again. The fifth sump has not been explored yet. Diving explorations in other direction, from the terminal lake of Planinska jama, have revealed a large flooded gallery that has been explored to the length of 500 m. Its cross-section reaches 15 m x 20 m and it sinks more than 50 m below the level of the lake. The straight line distance between two extreme points of explored channels in Pivka jama and Planinska jama is about 800 m. The complete vertical extent...
of the Postojna-Planina system is 115 m. Connection proved by explorer is expected anytime.

**BASIC CHARACTERISTIC OF STAGE DYNAMICS AT OBSERVED LOCATIONS**

In Fig. 3, time series for stage, temperature (and inflow for the Pivka Ponor) are shown for the seven sample sites. These data have been collected during Mar 2008 and Feb 2009 with Schlumberger Divers, recording at 10 to 15 minute intervals (see Turk (2010) for detailed description).

1. Ponor of Pivka river and the entrance hall (Dom): While summer recharge during the campaign has been fairly low (generally below 10 m$^3$/s), two episodes with stronger recharge appear: Mar–May 2008 (up to 30 m$^3$/s) and Dec 2008 to Jan 2009 (up to 60 m$^3$/s). During extreme floods the ponor area gets completely flooded.

2. Lower Tartarus: Stage in this part of the cave follows both in amplitude and in recession the flood pulses recorded at the ponor. Peaks are rather sharp (up to 2 to 3 m), and recession limbs very short, thus flow is rushing through an open channel, except at very high flow conditions.

3. Otoška Jama: Here, flood pulses result in a stronger increase in stage (up to 6 m, and occasionally more than 10 m), and flood recedes much slower. This behaviour indicates a more restricted flow regime, caused by the breakdown further downstream.

4. Martel Breakdown: Directly before the Martel rockfall, water can pond up to 8–10 m during high-flow conditions. The pronounced recession limbs indicate the slow drainage of these flood pulses through the rockfall.
5. Martel Chamber: Water level in Martel’s Chamber is rather complicated, reflecting the various flow paths possible during different flow regimes. While the phreatic passage carrying of base flow seems to be rather small, flow above 5 m³/s causes ponding and drainage through an upper gallery in about 3 m height. This ponding remains for long times, as the lower phreatic loop is too small to efficiently drain the flooded chamber. At very high flow rates, ponding is even more pronounced, before another overflow in 10 m height is activated.

6. Magdalena jama: Stage variations in Magdalena jama are rather small (1−2 m except for very large floods), which might be a result of water retention in Martel’s chamber. The two phreatic loops do not significantly limit flow.

7. Pivka jama: The large channel in Pivka jama is able to transmit water almost always under open-flow conditions, with changes in stage up to 4 m during normal floods, a short recession limb, and significant flooding only during large flood event an in december 2009. The latter flooding is changed either by the limited cross-section of the first bathy-phreatic passage, the sump reaching 17 m below the open channel, or later constrictions.

SwMM MODEL OF THE PIVKA CHANNEL

We have based our layout for the active conduit system of Postojnska jama on survey data. The main active system, comprising epi-phreatic and phreatic passages from the Pivka Ponor to the known terminal sump in Pivka jama, has been discretised in map view (UTM coordinates) to obtain junction nodes for SwMM. The conduits connecting these junction nodes have properties derived from cave cross sections, with large epi-phreatic passages modelled with rectangular cross sections and small by-passes such as the Martel rockfall and the Martel Chamber as well as the sumps as circular conduits.

The resulting model is shown in Fig. 4 as simplified cross section. The model starts with rectangular conduits from Pivka Ponor (1) to Martel’s rockfall (4), generally 2 m x 15 m in size (width times height), with a narrowing of passage width to 1 m just before Otoška jama (3). Martel’s rockfall (4), which blocks almost the entire active passage, is by-passed by four circular conduits in 0, 3, 5, and 10 meter height above the bottom of the passage (elevation offsets from cave survey). Diameters for these four conduits are variables (a1, a2, a3, a4). Between Martel’s rockfall (4) and Martel Chamber (5), the active passage is again large (2 m x 10 m, rectangular cross section), but Martel Chamber itself is modelled as constriction with three circular conduits in 0, 3, and 10 m height above the bottom of the passage. The lowest by-passing conduit resembles the phreatic base flow (unexplored), the two other conduits become active only at higher flow rates. Conduit diameter are variables (b1, b2, b3). After Martel Chamber the passage is large again (2 m x 10 m, rectangular cross section) in Magdalena jama (6), with the exception of the sump, modelled as circular pipe down to −17 m and diameter taken as variable (c1). In Pivka jama (7), passages are large (2 m x 10 m, rectangular cross section), except the two sumps, with the first one being a flow constriction, leading down to −28 m and diameter as a variable (d1, d2).

The SwMM model continues all the way down to Planinska jama (not shown in Fig. 4, but all conduits are modelled big enough to transfer incoming way under open-flow conditions to the karst spring. Again, cross sections for Planinska jama are based on the cave survey, only the unexplored passage between the end of Postojnska jama and the end of the Pivški rokav (Pivka branch) in Planinska jama is estimated (as circular conduit).

Manning coefficient used in all models was 0.01 and was not considered as an varying parameter, as this parameter is coupled to passage geometry.
In this section, we present our modelling results for the Pivka channel in Postojnska jama in two steps.

**MODELLED STAGE DATA**

In Fig. 5, the modelled stages for the cave sites Lower Tartarus (2) to Pivka jama (7) are shown. The SWMM model is driven calibrated to the Mar-May 2008 discharge record at Pivka Ponor, applied as flow rate into the conduit system (see Turk 2010 for details). Shown are both the observed stage (red lines) and the modelled stage (blue lines) at all six cave stations. The parameter values are set to the best-fitting parameter values (see Tab. 1) as a result from an inverse estimation, described in the next section.

The overall fit of the SWMM model results is very satisfactory (rms between 2–3), as both the amplitudes and the recession limbs of the flood pulses are well fitted. Some minor offset as visible in the modelled response for the Otoška jama site could become even better, if the width of this model section will be narrowed slightly. The only problem fitting the shape of the stage time series occurs in Martel Chamber: Here several of the flood events, which result in a slow recession recession of stage at this location in the data, cannot be properly fitted, as the

![Graph showing modelled stages for various locations in Postojnska jama](image.png)

*Fig. 5: Data and model for stage at seven locations in Postojnska jama for the period Mar-May 2008. The time is given in days starting on 1. March 2008. Shown is observed stage (red lines) and modelled stage (blue lines) for all seven locations.*
modelled stage response is too fast. On the other hand, the modelled stage can fit the observed stage for higher flow rates resonably well. Obviously, the characteristics of flow geometry of Martel Chamber would need some more adjustment.

STATISTICAL ANALYSIS
We have performed an inverse analysis of the prediction, using a neighborhood algorithm for creating and analysing a set of forward calculations.

The Neighbourhood (NA) algorithm (Sambridge 1999a, b) is a direct search method for nonlinear inverse problems. The NA approach is applicable to a wide range of inversion problems, particularly those with rather complex dependencies between data and model. During the search stage of the NA algorithm, a multidimensional parameter space is sampled for combinations of model parameters, which provide a satisfactory fit to the observed data. The search is guided by randomised decisions similar to techniques used for genetic algorithms (GA) and simulated annealing (SA), but the NA algorithm needs only two control parameters. A misfit between model prediction and observation is calculated, and the search is driven towards the minimum misfit within the parameter space. The NA algorithm is based on the geometrical concept of Voronoi cells. These Voronoi cells are nearest-neighbour regions around each sampling point. The Voronoi cells are used to guide the sampling (see Sambridge 2001, for more details).

The fitting criterion used to assess the model quality is the root-mean-square (rms) fit:

\[
    rms_f = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{h_{obs,i} - h_{pred,i}(x_i)}{h_{err}} \right)^2}
\]

(7)

Here, \( h_{obs,i} \) and \( h_{pred,i} \) are observed and predicted stage values from all stations and all sampled times (counter \( i=1,n \)), \( x_i \) is the parameter value vector (our free parameter values), and \( h_{err} \) a stage uncertainty set to \( h_{err}=10 \) cm.

As free parameter values we choose the lower conduits in Martel’s rockfall and Martel Chamber (\( a_1, a_2, b_1 \)), the diameter of the Magdalena and the Pivka sumps (\( c_1, d_1 \)), and the passage width of the cave section before Otoška jama and the sump of Pivka jama (\( w_1, w_2 \)). Other parameter values are fixed (see Tab. 1). Sampling all free parameters at once would result in too many calculations, as a single forward run of SWMM5 takes about 5–10 minutes. We have therefore adopted the following strategy:

(i) First obtain a good-fitting forward model (see initial values in Tab. 1). Based on the parameter values of this first run,

(ii) determine \( a_1, a_2, b_1 \) by inversion, while keeping the other values fixed,

(iii) determine \( c_1, d_1 \) by inversion, while keeping \( a_1,a_2,b_1 \) to their best fit and the other values fixed,

(iv) determine \( w_1, w_2 \) by inversion, while keeping the other values to their best fit.

Parts (ii)-(iv) are called sub-sets in the following discussion. This choice allows us to discuss the importance of three sets of parameters, firstly the by-passes in Martel’s rockfall and Martel Chamber (\( a_1, a_2, b_1 \)), the diameter of the Magdalena and the Pivka sumps (\( c_1, d_1 \)), and the passage width of the cave section before Otoška jama and the sump of Pivka jama (\( w_1, w_2 \)). Other parameter values are fixed (see Tab. 1). Sampling all free parameters at once would result in too many calculations, as a single forward run of SWMM5 takes about 5–10 minutes. We have therefore adopted the following strategy:

(i) First obtain a good-fitting forward model (see initial values in Tab. 1). Based on the parameter values of this first run,

(ii) determine \( a_1, a_2, b_1 \) by inversion, while keeping the other values fixed,

(iii) determine \( c_1, d_1 \) by inversion, while keeping \( a_1, a_2, b_1 \) to their best fit and the other values fixed,

(iv) determine \( w_1, w_2 \) by inversion, while keeping the other values to their best fit.

Parts (ii)-(iv) are called sub-sets in the following discussion. This choice allows us to discuss the importance of three sets of parameters, firstly the by-passes in Martel’s rockfall and Martel Chamber, secondly the sumps, thirdly the passage constrictions before Otoška and Pivka jama. We have tested the influence of fixing the first sub-set on the results for the third sub-set, and found only very small effects for our final model parameter values.

For all three subsets, we run 100 models first, then pick the best two model runs (lowest rms values), resample each of them 10 times, and repeat the resampling four times. This results in 140 models altogether for each subset. The range for each model parameter is given in Tab. 1.

<table>
<thead>
<tr>
<th>Location</th>
<th>Diameter</th>
<th>Unit</th>
<th>Initial value</th>
<th>Range</th>
<th>Best value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Otoška jama</td>
<td>w1</td>
<td>M</td>
<td>1.00</td>
<td>0.50-2.00</td>
<td>0.88</td>
</tr>
<tr>
<td>Martel’s rockfall</td>
<td>a1</td>
<td>M</td>
<td>1.25</td>
<td>1.00-1.50</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>a2</td>
<td>M</td>
<td>1.25</td>
<td>1.00-1.50</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>a3</td>
<td>M</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>a4</td>
<td>M</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Martel’s Chamber</td>
<td>b1</td>
<td>M</td>
<td>1.00</td>
<td>0.75-1.25</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>b2</td>
<td>M</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>b3</td>
<td>M</td>
<td>3.00</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Magdalena sump</td>
<td>c1</td>
<td>M</td>
<td>4.00</td>
<td>2.00-6.00</td>
<td>(5.28)</td>
</tr>
<tr>
<td>Pivka jama</td>
<td>w2</td>
<td>M</td>
<td>1.50</td>
<td>0.50-2.00</td>
<td>1.35</td>
</tr>
<tr>
<td>Pivka sump</td>
<td>d1,d2</td>
<td>M</td>
<td>2.500</td>
<td>2.00-4.00</td>
<td>(3.90)</td>
</tr>
</tbody>
</table>

Tab. 1: Model parameter ranges and best-fit parameter values for constrictions in the SWMM model for Postojnska jama.
As a confidence parameter, we calculate

$$\psi_f = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{h_{\text{pred},i}(x_{\text{best}}) - h_{\text{pred},i}(x_f)}{h_{\text{err}}} \right)^2}$$

with $h_{\text{best},i}$ and $h_{\text{pred},i}$ the best-fitting and the other predicted stage models. $x_{\text{best}}$ is the parameter value vector for the best-fitting model and $x_f$ is the general parameter value vector. The $\psi$-confidence parameter reports the goodness-of-fit with $\psi<2$ for models comparable with the best-fit model within the 2$\sigma$-uncertainty, and $\psi<1$ for models comparable with the best-fit model within the 1$\sigma$-uncertainty.

In Fig. 6, the fitted model parameter values are shown for the three subsets. The figure describes, for each of the parameter values, all calculated models (light gray), the best-fitting model (red dot), all models similar to the best-fitting models with the 2$\sigma$ (dark gray) and 1$\sigma$ (black) confidence range.

For the first subset ($a_1$, $a_2$, $b_1$ free, other parameter values fixed), we obtain a narrow range of possible models. As these tested parameter values control the lower by-passes in Martel's rockfall and Martel Chamber, we conclude that these flow constrictions are very important for fitting the stage observations, as they control both throughput and ponding.

Moving to the second subset ($c_1$, $d_1$ free, other parameter values fixed) describing the diameter of the two sumps, the models fitting within the 1$\sigma$-confidence range cover a wide range, indicating the secondary importance of the sump constrictions (if they are not too small, of course). This results clearly indicates, that the constrictions upstream of the sumps are controlling the flow, and the sumps are large enough to pass the water from Martel Chamber and Pivka Channel without delay further downstream.

The last subset ($w_1$, $w_2$ free, other parameter values fixed) controls flow through the passage constrictions in Otoška jama and Pivka jama, setting the height of the stage in these sections. Again, we find a small range for the two width parameters to obey the 1$\sigma$-confidence. This again explains the importance of passage constrictions to the stage recorded in the cave.
Fig. 7: Misfit as a function of inverse iteration counter for all inverse models. In the legend, model parameters with subscript only are inverted parameters, model parameters in brackets are fixed parameters, model parameters with astericus as superscript are fixed best-fit parameters. Note the logarithmic scale for RMS.

Fig. 8: Data and model for stage at seven locations in Postojnska jama (data from Turk 2010) for the period May-Jul 2008. The time is given in days starting on 1. July 2008. Shown is observed stage (red lines) and modelled stage (blue lines) for all seven locations.
The observation of stage at seven locations along the active flowath of the underground Pivka River through Postojnska jama enables us to study both temporal but also spatial changes, while flood events traverse the cave. The results discussed above are confirmed while looking at the rms value as it changes throughout the inversion (Fig. 7). For the first subset, rms values are varying about one order of magnitude during the initialisation phase (first 100 models) of the inversion process, then the models quickly converge to the best-fit model \((rms=3)\). For the second subset, which is started with fixing the values for \(a1\) and \(a2\) to the best-fit parameters just obtained, there is no real improvement in the inversion, because the diameter of the sumps is no real constriction. However, the third subset again results in a large variation of rms values for the initial phase, then it converges quickly to the best-fit value.

INDEPENDENT TEST OF BEST MODEL
To test the significance of the SWMM-model result describing the active galleries in Postojnska jama, we apply the model with its best-fitting parameter distribution to the observed stage data from Pivka ponor for a different period from May to July 2008 (see Fig. 3). The resulting stages for the sample sites are shown in Fig. 8: This period is characterised by three moderate flood events, which are fitted reasonably well at all locations, except for Martel Chamber, whose flow characteristics again are not properly captured by the SWMM model. Another interesting feature is the stage record in Magdalena jama: Here, the stage data for the first 60 days are well fitted, but from then on a significant underestimation of stage appears, when compared to the observed stage data. The observed stage data remain high after day 65, without presence of a real flood event. We might speculate a problem with the diver.

LOOKING FURTHER DOWN TO PLANISKA JAMA
The entire hydraulic model presented stretches from the Pivka sink to the spring of Planinska jama, however we assessed only the upper part describing the Postojnska jama cave system, as here both the cave geometry is well known and the seven sample sites provide good spatial coverage throughout the active cave system.

Nevertheless, water passing through Postojnska jama reappears in Planina Spring, and thus we can compare the outflow through our model with the discharge of the Unica River, monitored at Hasberg station about 1 km down from the spring. In Fig. 9, both the observed (red line) and the predicted (blue line) discharge is shown for the period March 2008 to July 2008. The observed discharge has a large variability, from \(2-3\ m^3/s\) during low-flow conditions, to \(90\ m^3/s\) during large floods. Recession limbs are rather pronounced. The modelled discharge through the Pivka Branch of Planinska jama, one of the two active rivers in Planinska jama, contributes less than 50 % of the total discharge, and flood peaks are more pronounced with sharper recession curves. The difference between the observed and the modelled discharge is the contribution from the Rak Branch of Planinska jama, carrying down the water from Cerkniško Polje via Karlovica, Zelške and Tkalca jama and contribution from Malenščica and intermittent Škratovka springs.

CONCLUSIONS
The observation of stage at seven locations along the active flowath of the underground Pivka River through Postojnska jama enables us to study both temporal but also spatial changes, while flood events traverse the cave.
The flow is mostly controlled by natural barriers such as rock falls, sumps, or passage constrictions. With our simplified cave geometry for hydraulic modelling of the flood pulses, we are able to estimate the hydraulic properties of these barriers. The hydraulic model has the potential to run as a predictive tool for flood forecasting within the cave system. Turk (2010) achieved a similarly good fit using machine learning algorithm. Such algorithms are powerful tools for flood predictions, but hydraulic models tell more about the nature of the system.

The work demonstrates that flood event hydraulics of a well developed telogenetic system can be modelled solely with conduit network models. Even more, the model geometry used here is very simple compared to the natural conduit system, but it turns out that only few restrictions dominantly control the hydraulics. It seems that finding such restrictions in any well developed karst system might be crucial for their successful characterisation and modelling.

ACKNOWLEDGEMENTS

The data presented in this work were collected in the scope of a PhD research by JT, financed by the Slovenian Research Agency. GK would like to thank the German Research Society (Deutsche Forschungsgemeinschaft - DFG) for funding the Project KA1723/6-2 within this work was prepared. We thank Franjo Drole for keeping us up-to-date with current explorations of the Postojna-Planina system. We would also like to thank to the group of divers and cavers, which are continuously unlocking the mysteries of this important cave system.

Furthermore, the constructive and very helpful reviews provided by two anonymous reviewers were of a great help when preparing this manuscript.

REFERENCES

Gabrovšek, F. & J. Turk, 2010: Observations of stage and temperature dynamics in the epiphreatic caves within the catchment area of the Ljubljanica River (Slovenia).- Geologica Croatica, 63, 2, 187−193. http://dx.doi.org/10.4154/gc.2010.16
Peterson, E. & C. Wicks, 2006: Assessing the importance of conduit geometry and physical parameters in karst systems using the storm water management model (SWMM).- J. Hydrol., 329, 295–305. http://dx.doi.org/10.1016/j.jhydrol.2006.02.017


