# PREFERRED NUMBERS IN MODULAR SIZES OF EMONA, DIOCLETIAN'S PALACE, AND MOGORJELO

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Numbers with special meaning were used not only in modular dimensions of Christian architecture<sup>1</sup> but also in the pre-Christian compositions. To illustrate their presence in sizes of Roman composition, the dimensional analysis of Emona, of Diocletian's Palace, and of Mogorjelo, is the topic of this article.

Numbers 2, 3, 5, 6, 7, 10, 12, 100, 127, 432, 532, and 666, in the above compositions, are usually called mystical numbers, but the preferred numbers would be a more suitable apellation, since their only mysticism is a Christian addition. Christianity has forgotten their original meaning. »Ignorance of numbers... prevents us from understanding things that are set down in Scripture in a figurative and mystical way« wrote the educated Saint Augustine.<sup>2</sup>

However, it would exceed the scope of this paper to deal with either Christian<sup>3</sup> or rational explanation of preferred numbers. The purpose of this paper is only the documentation of selected numbers in the modular composition of Emona, Diocletian's Palace, and Mogorjelo, and the description of their mathematical origin.

The length and breadth of Emona<sup>4</sup> are 360 and 300 passus. But the 360 passus length equals also

6 modules of 60 passus and 432 modules of 50 unciae. The 300 passus width is equal to

- 2 modules of 150 passus,
- 3 modules of 100 passus,
  - 5 modules of 60 passus,
- 6 modules of 50 passus,
  - 10 modules of 30 passus,
  - 12 modules of 25 passus,

100 modules of 3 passus, and

532 modules of 45 digiti, if a 0,25 percent approximation is allowed. Besides, the diagonal of Emona, which is the longest extension of the composition, equals 127 modules of 666 : 3 *unciae* or

666 modules of 127:3 *unciae* with a 0,27 percent correction.

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The Diocletian's Palace<sup>5</sup> is 29,5 modules of 5 passus long on the eastern side, whereas its western side is slightly longer, due to the irregularity of the plan. The length of the eastern side is also 531 modules of 200 : 3 sicilici; since the western side is longer, it can be said that average Place's length is

532 modules of 200: 3 sicilici. The Palace's northern width is 24 modules of 5 passus which equals also

532 modules of 200 : 3 sicilici. The Palace's northern width is 24 modules of 5 of 5 passus long which equals

2 modules of 15.000 sicilici

3 modules of 10,000 sicilici

5 modules of 6.000 sicilici

6 modules of 5.000 sicilici and

10 modules of 3.000 sicilici. It seems that the southern width and the western length of the Palace were made longer to get incorporated more symbolic numbers, and not because of an obstacle of an unusually hard rock or because of the Roman inability to construct a right angle. This type of composition was obviously regarded as an especially fortunate solution because of the good omen in its dimensions, since the same proportional deformation was used more than once. The atrium of the Roman villa in Fishbourne, e.g., is dimensionally analogous to the plan of Diocletian's Palace.6

The length and width of Mogorjelo<sup>7</sup> are 70 and 60 passus, respectively. 70 passus equals

7 modules of 10 passus,

10 modules of 7 passus and

127 modules of 44 digiti with a 0,21 % correction. The 60 passus equals also 6 modules of 10 passus,

10 modules o 6 passus and

532 modules of 27 sicilici (or 9 digiti) with a 0,25% deviation.

Here is a short description of the listed preferred numbers:

2 is regarded as a female number by Greeks; 3 is a male number:

3 is a male number:

5 is the number of love;

6 is an Euclidean perfect number;

7 is the Jewish holy number and one of the four Moslem complete numbers (kāmil); besides, numbers 3 and 7 are Tolstovan masonic numbers;

10 is a Pythagorean perfect number or tetractys; it is the basis of the decimal system;

12 is one of the kāmils and the basis of the duodecimal system;

100 is a square of an already perfect 10;

127 is the diameter of Critias' Atlantis;

432 is the basis of the Indian chronometry;

532 is the year count of the solar-lunar cycle;

666 is the biblical number of the Beast.8

The smaller integers are obvious. They are used because of their specific compositional properties.9 But the larger numbers are not so obvious, though they seem to

have a perfectly logical *raison d'être*. Mathematically, they originate in the scheme of the Vitruvian man-in-the-circle-and-the-square<sup>10</sup> popularly known through the Leonardo's drawing.

The Vitruvian man has been studied by two Scandinavian researchers. Eivind Lorenzen derived from it the geometrical series with the coefficient  $\sqrt{5}$  and Thomas Thieme added the  $\sqrt{3}$  series.<sup>11</sup> Both series start with the 1 orgyia long side of the man's square, which is equal, according to them, to

- 4 royal cubits,
- 8 pedes,
- 20 natural hands,
- 22,4 natural fists.
  - 25 derived hands,
  - 28 derived fists,
  - 100 fingers,
  - 112 single inches,

128 double inches. The above numbers together with coefficients  $\sqrt{5}$  and  $\sqrt{3}$  produce two series of irrational numbers. I rationalized the irrational geometric terms by making them whole. The result was two arithmetical series composed of preferred numbers, which can be found as modular multiples in dimensional composition of ancient architectures, in music as the vibration numbers of preferred tones, and in diameters of Atlantis after Critias<sup>12</sup> which is a model of the heliocentric Universe<sup>13</sup>, as modular multiples.

Besides, the scheme of the Vitruvian man appears to be the set of Platonic or Chinese elements, which were the paradigm of such eastern architectural forms, as the Indian *stupa* and the Moslem mosque.

It can be said that the Western architecture followed the rational arithmetic way whereas the eastern architecture adopted the irrational geometrical form of the same mathematical scheme, known today as Vitruvian man or the Chinese elements.<sup>14</sup>

<sup>1</sup> Leone Batista Alberti, Ten Books on Architecture (A. Tiranti, London 1951), Book IX, Chapter V. — E. Read Sunderland, Symbolic Numbers and Romanesque Church Plans, Journal of Society of Architectural Historians 18 (1959) pp. 94—103. — K. J. Conant, Les dimensions systématiques et symboliques à l'église abbatiale de Cluny, Annales de l'Académie de Mâcon, 3<sup>e</sup> série, tome XLV, Mâcon 1960—1961. K. J. Conant, Cluny, Cambridge, Massachusetts, The Mediaeval Academy of America, Mâcon 1968; Chapter: Dimensions systématiques et symboliques. K. J. Conant, Speculum, Cluny Studies 1968—1975, A Journal of Mediaeval Studies, vol. L no. 3, July 1975. T. Kurent, Cosmogram of the Romanesque Basilica at Stična, Univerza v Ljubljani, Ljubljana 1977.

<sup>2</sup> Works of Augustine, On Christian Doctrine, II, 25. <sup>3</sup> An expert in holy scriptures has already expressed his dissatisfaction with my omission to quote Christian fathers and with my preference to look for the practical compositional properties of preferred numbers in my *Cosmogram*.

<sup>4</sup> M. Detoni, T. Kurent, *The Modular Re*construction of Emona, Razprave narodnega Muzeja v Ljubljani, Ljubljana 1963.

<sup>5</sup> T. Kurent, The Modular Composition of Diocletian's Palace in Split, *Živa Antika* 20 (1970) pp. 167—170. T. Kurent, Proportio and Commodulatio after Vitruvius Compared to Proportion and Modules of Diocletian's Palace in Split, *Živa Antika* 21 (1971) pp. 217—230.

<sup>6</sup> T. Kurent, The Modular Analogy of the Roman Palaces in Split ind Fishbourne, *Archaeometry* 12 (1969) pp. 37–42.

<sup>7</sup> T. Kurent, The Analogy in Modular Composition of Roman Fortresses at Carleon and at Mogorjelo, Živa Antika 20 (1971) pp. 659—662.

 <sup>8</sup> Revelation 13, 18.
<sup>9</sup> cf. T. Kurent, Vloga števila 7 v modularni kompoziciji, Arheol. vestnik 13-14 (1962-63) 529.

 <sup>10</sup> Vitruvius, De Architectura, I. I. 3 sq.
<sup>11</sup> E. Lorenzen, Technological Studies in Ancient Metrology, Nyt Nordisk Forlag, Arnold Busck, Copenhagen 1966. E. Lorenzen, »along the lines where columns are set«, Nyt Nordisk Forlag, Arnold Busck, Copenhagen 1970. Th. Thieme, Montecassino: An Example of Planning in the Vitruvian Circle, Opuscula Romana XI: 10, Acta Instituti Romani Regni Sveciae, Stockholm 1976. Th. Thieme, J. Beck, La cattedrale normana di Cefalú, Analecta Romana Instituti Danici VIII, Odense University Press, 1977. <sup>12</sup> Plato, *Critias*, 115 E—117 E.

<sup>13</sup> T. Kurent, The Platonic Atlantis is a Scale Model of the Heliocentric Universe. - Prepared for the Architectural Association Quarterly, London.

<sup>14</sup> T. Kurent, The Vitruvian Man in the Circle, the Five Platonic Elements, and the Preferred Numbers in Ancient Architecture. — In preparation.

## Illustration 1

There is a difference between our abstract numbers and the ancient figured numbers,

made of pebbles and arranged in different shapes. Calculi forming triangles, squares, cubes, etc., are easy to visualize and ofter a good exercise in the logic of composition. The forms of numbers are stimulating the eidetically gifted. Besides, they have a mnemonic function.

There is no limit of optical analogies for a vivid imagination. Odd numbers of pebbles, e. g., arranged in a triangle similar to the shape of public region are said to be male numbers; the split triangle of even numbers is evidently composed of female numbers. Consequently, the first female and the first male number, 2 and 3, form the number of love, i.e. 5, which is called γάμος, or the number of Aphrodite.

The figured numbers are a visual form of whole numbers which suits the logic of modular architectural composition. Number of building components in a composition is a whole number. A column, a beam, a building block per se, is a whole; their aliquot parts can not be used in a composition; their modular sizes are equal to whole multiples of a module; there is no such a thing as a half-module; every module, from the smallest to the largest, is a monade, a unit in its own right.

Broken blocks and other amorphous building parts are not modular components, but they can be used as filling material for construction, of a wall, e.g.

One pebble is a whole, a unit, which can represent not only what we call number one, but also a legion of ten cohorts, a dozen of eggs, a day of twenty-four hours, a decade, or any other composed unit. It is a great simplification in substituting one pebble (meaning 1 legion, e.g.) for so many constituent parts (cohorts or even legionaires, e.g).

One half of a composed unit is a whole number of constituents; a detachment of an uneven number of soldiers is to be split in two unequal halfs if the poor odd soldier is to be spared.

The multiform meaning of a figured number is probably responsible for the understanding, that a Pell term signifies not only itself but also its synonymous numbers, which are 2, 4, 8,... and/or 10, 100, 1000 ... times larger or smaller.



HEXAGONAL	NUMBE	ERS	
n(4n-2) 2 GNOMON	1,	6, 15	
(4n-3)	· 1,	5, 9,	
HEPTAGONAL	NUME	BERS	
<u>n(5n-3)</u> 2	1,	7, 18,	
GNOMON (5n-4)	1, 1	6, 11,	
POLIGONAL	NUMBE	RS	
n(X(n-1)-2(r 2 GNOMON X(n-1)-(2n-3	) )		
CUBIC NUM	BERS		
n <sup>3</sup>	1, 8	8, 27,	
GNOMON 3n <sup>2</sup> -3n+1	1, 1	7, 19,	
TETRAHEDRA	L NUM	BERS	
n(n+1)(n+2) 1·2·3	1, 4	, 10,	
GNOMON			
<u>n(n+1)</u> 2	1, 3	3, 6,	
PYRAMIDAL	NUMBE	RS	
n(n+1)(2n+1)	- 1, 5	5, 14,	
3.2.1			
GNUMUN			

gr. ή ψηφος ~ lat. calculus

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The vitruvian man-in-the-circle, popularly known in the drawing by Leonardo, is, according to E. Lorenzen, a scheme of sizes, ranging from the 6 foot fathom of 72 inches to the 128 »double inches« orgyia.

The Lorenzen's divisors of the fathom and orgya are 72, 96, 100, 112, and 128. They are starting terms of geometrical series with coefficients from the helix irrationalium. The resulting terms are irrational, but should be rounded in integers. Among them, I have found that the most important preferred numbers from the man-in-the-circle scheme in ancient architecture are, besides 3 and 7, numeri, or better numerus, 111, 108, and 127. It is not easy to grasp that the above numbers are only various manifestations of the same numerus, if we do not underxstand that in figured numbers one pebble can symbolize any other group of pebbles. A Pell term is equal only to related numbers which are 2, 4, 8... and/or 10, 100, 1000... times langer or smaller, but a figured number equals also numbers are only manifestations of the same numerus. Subsequent divisions of 72, 96, 100, 128, 3, 7, 111, 108, and 127, with the coefficient 2, with the rounding of terms in smaller integers, prouves, that the end result of such a procedure is the monad:

72, 36, 18, 9, 4, 2, 1 96, 48, 24, 12, 6, 3, 1 100, 50, 25, 12, 6, 3, 1 128, 64, 32, 16, 8, 4, 1 3, 1 7, 3, 1 111, 55, 27, 13, 6, 3, 1 108, 54, 27, 13, 6, 3, 1 127, 63, 31, 15, 7, 3, 1 This before the second se

This baffling mathematical way of thinking leads to mysticism. The Christian monotheism equates God with the Trinity. In dualism, the two are one. In polytheism, the initial principle dissolves into the whole pantheon. But the strict moslem monotheism, expressed in the doctrine *Allah wahed ahed* — Allah is one and the only, maybe helped the development of the practical mathematics in Arab world.

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ο δργυιά 01 01 03  $\frac{128\sqrt{3}}{2} = 110,85125... \rightarrow (111)$ 128 DOUBLE INCHES  $\frac{100\sqrt{5}}{2} = 111,86339... \rightarrow 112$   $\frac{96\sqrt{5}}{2} = 107,33126... \rightarrow (108)$ 100 FINGERS  $\frac{96\sqrt{7}}{2} = 126,99606... \rightarrow (127)$ 96 DAKTYLO

The vitruvian man-in-the-circle-and-the-square is, according to E. Lorenzen, a scheme of preferred sizes; the circle's diameter or the square's side, equals one orgyia which can be divided either with 100 fingers, or 128 double inches, or 112 single inches. The listed sizes are interrelated:

100  $\sqrt{5}: 2 = 111,805 \sim 112$ , but

 $128 \sqrt{3}: 2 = 110,85 \sim 111.$ 

The 111 single inches orgyia was introduced by Th. Thieme with his  $\Delta$  canon. Geometrically, the same can be achieved with an equilateral triange inserted in the scheme. I have expanded the scheme by Lorenzen and Thieme further: Another equilateral triangle, inserted in the oppozite side of the square, and the Thieme's triangle intersect in the form of a losenge. An arc with centre at the middle of the square's base and the discrete the triangle intersect. radius stretching to the opposite tip of lozenge cuts the circle and forms with it a crescent.

The forms of the man-in-the-circle scheme, i.e. the square, circle, triangle, crescent, and lozenge, are symbols of the platonic elements.

It seems that elements the Cosmos after Plato is made of are an architectural tool forming the man-made world.



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The orgyia of the man-in-the-circle is subdivided in 72, 96, 100, 111, 112, and 128, parts. Among the terms of series, starting with the above numbers, having the coefficient  $\sqrt{3}:2$  and  $\sqrt{5}:2$ , the same numbers reappear. Or, more precisely, some of the irrational terms can be rounded in the preferred integers, 72, 96, 100, 111, 112, 128, and additional terms 40, 41, and 42.

If we remember that synonymous numbers are 2, 4, 8... and/or 10, 100, 1000... times larger or smaller, it is evident, that the number 666 results in 41:

666:2=222

 $333:2 = 166.5 \rightarrow 166$ 

166:2=83

 $83:2 = 41,5 \rightarrow 41.$ 

The analogous subdivision with the coefficient 3 makes the number 666 equal to 111. It is interesting that the vibration numbers of the natural harmonical tones are the same numbers: 72, 96, 128, 432,  $41 \sim 42$ .

Besides, the modular multiples of Atlantis rings according to Critias are synonymous to preferred numbers:

100, 40, 112, 532, 729, 72, 96, 42, 432, and 127. Atlantis is a precise model of our heliocentric universe and the preferred numbers in the orbital diameters are the origin of the pythagorean musical spheres. But, this story is too long to be repeated here. See T. Kurent, *The Cosmogram of the Romanesque Basilica at Stična*, Fakulteta za arhitekturo, Univerza v Ljubljani 1978.



MA STRUCTURA VITRUVI CE ARI



Emona, an augustean colony under the present-day Ljubljana, is 360 passus long and 300 passus wide. Its planning module is 60 passus long and its plan is in the ratio 6:5, which is a rational approximation of the proportion, called quadriagon. Numbers 5 and 6 are consecutive terms in the first Pell series:

1 2 5 12...

6

The preferred numbers in the dimensional composition of Emona are following: 300 passus width equals 2 modules of 150 passus

3	modules	of	100	passus	
5	modules	of	60	passus	
6	modules	of	50	passus	
10	modules	of	30	passus	
12	modules	of	25	passus	
100	modules	of	3	passus	
532	modules	of	45	digiti.	

permitted:

6 modules of 60 passus or 360 passus length equals 432 modules of 50 uniciae;

the diagonal of the  $360 \times 300$  passus plan equals

127 modules of 666:3 unciae or

666 modules of 127:3 unciae if the approximation with the difference of 0,27 % is permitted.

if the tollerance of +0.25 % is

More about Emona in the book

M. Detoni and T. Kurent, Modularna rekonstrukcija Emone — The Modular Reconstruction of Emona, Narodni muzej, Ljubljana 1963.



TURRIUM MURORUMQUE FUNDAMENTA SIC SUNT FACIENDA, UTI FODIANTUR, SI QUEAT INVENIRI, AD SOLIDUM ET IN SOLIDO, QUANTUM EX AMPLITUDINE OPERIS PRO RATIONE VIDEATUR, CRASSI TUDINE AMPLIORE QUAM PARIETUM, QUI SUPRA TERRAM SUNT FUTURI, ET EA IMPLEANTUR QUAM SOLIDISSIMA STRUCTURA. VITRUVII DE ARCHITECTURA LIB. 1.75



The Diocletian palace in Split, Dalmatia, is theoretically 24 modules wide and 29 modules of 5 *passus* long. Numbers 24 and 29 are terms of the first Pell series:

1 2 5 12 29 24

The ratio 29:24 is close to the proportion of *Quadriagon*. But the theoretical sizes are adjusted either because of the proportional, or because of numerological, or perhaps, because of both, reasons: The northern elevation is 24 M (5 passus), or 120 passus, long, but the southern elevation is one module longer. It is 125 passus long. The theoretical length of the western elevation is 29 modules, but because of the correction, its actual length is 5 gradus longer. Its length is 147,5 passus.

The eastern elevation is slightly longer because of the larger southern width and larger than 147,5 *passus*.

> 147,5 passus length results in 532 M (66,6 sicilici). 120 passus length equals 432 modules of 66,6 sicilici, 125 passus length equals 2 M (15.000 sicilici)

3 M (10.000 sicilici) 5 M ( 6.000 sicilici) 6 M ( 5.000 sicilici) 10 M ( 3.000 sicilici)



Mogorjelo is a villa fortificata in Hercegovina on the border with Dalmatia. It is 6 M (10 passus) in width and 7 M (10 passus) in breadth. Ratio 7:6 is a standard proportion, called quadriagon, from the second Pell series:

1 3 7 17 ... 6

The width of 60 passus equals also

6 M (10 passus),

10 M (6 *passus*), and 532 M (27 *sicilici*) with 0,25 percent correction.

The length of 70 passus is equal to

7 M (10 passus),

10 M (7 passus), and

127 M (44 digiti) with 0,25 percent correction.

In the world where only integers are known, the rounding of figures in whole numbers is not unusual.



MODULE M 10 PASSUS

0 5 10

PELL SERIES  $1 - \theta - \theta^2 - \theta^3 - \theta^4 - \theta^5 - \cdots$   $1 - 3 - (7) - 17 - 41 - 99 - \cdots$  $2 - (6) - 14 - 34 - \theta^2 - 198 - \cdots \rightarrow \lim \frac{\theta}{2}$ 

## PROPORTIO

 $\frac{7}{6} \sim XD$  (OUADRIAGON)  $\sim \frac{\theta}{2} = \frac{\sqrt{2}+1}{2} = 1,207... \simeq 1,166 ... = \frac{7}{6}$ 



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PASSUS

The forms of the four platonic elements — earth, water, fire, and air — shape a typical mosque, here represented by the Gazi Husrevbey mosque in Sarajevo, Jugoslavia. The forms of the five Chinese elements — earth, water, fire, metal, and wood — shape a typical Buddhist stupa.



Podunavije u osmom i devetom vijeku. Naprotiv u našite poluzerounizana stol takinama se satrai sporev stavij situ stavani prati materijal iz XII do XX vijeka. Zato spatrano da su sve oblike za danisti šimosi je nase omobine tožno žadostvavlje ali esta četom i rednine vijeku dotavijeu konstit estalerot uzurtim polučar jevijej da su jeve od četom i rednine vijeku dotavijeu konstit estalerot uzurtim polučar jevije sa roče od četom i rednine vijeku dotavijeu konstit estalerot uzurtim polučar jevije sa roče od četom i rednine vijeku dotavijeu konstit estalerot uzurtim polučarna jevije sa roče od četom i rednine vijeku dotavijeu konstitu stale i stalim polučarna jevije sa roče od četom i rednine vijeku polača polača police u stale sto ze tice razvoje u opanih sastanje tradanje u jestav jevi dotavije vred sto ze tice razvoje u opanih sastanje tradanje u jestava jevi tera od vije u traja storije kolice jevijeka polici dotavi stola dotavije je na dotava jevije u traja i store storije kolice jevijeka polici dotavi do dotavije store u store i store i polite u polici već storije kolice nastavije u opanih sastanje u porednju store i traja i tr

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