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ANALYSIS OF HIGH WATERS ON THE KRIVA REKA RIVER, MACEDONIA

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The Kriva Reka near the Trnovec gauging station.

Analysis of high waters on the Kriva Reka River, Macedonia

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ABSTRACT: The Kriva Reka River is located in northeastern Macedonia. Only a few manmade facilities, such as canals, dams, embankments, and hydropower plants, have been built along the river. This river type is particularly useful for calculating high waters using mathematical and statistical methods. To this end, five theoretical distributions were used in this study: the Gaussian normal distribution, log-normal distribution, Gumbel distribution, Pearson type III distribution, and log-Pearson type III distribution. In order to determine the probability of the occurrence of high waters at the Trnovec gauging station on the Kriva Reka, a period of 39 hydrologic years was processed, each year beginning in October.

KEY WORDS: geography, hydrology, Kriva Reka, high waters, maximum annual discharge, distributions, probability, Macedonia.

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1 Introduction

Rock composition, relief, vegetation, and precipitation are the main factors influencing the development of high waters in the Kriva Reka basin. The basin is largely composed of low-permeability volcanic and metamorphic rocks. The river valley has a gorge character with a small percentage of forest. Precipitation is especially high in spring and autumn, and the river is subject to flash flooding. Historical floods in the Vardar River basin occurred in 1778, 1876, 1895, 1897, 1916, 1935, 1937, and 1962 (Sibinović 1968).

The Kriva Reka is a left tributary of the Pčinja River, which is a left tributary of the Vardar River. The river is part of the Aegean watershed area. Its specific climate is a blend of continental and Mediterranean influences, providing a particular river regime formed by pluvial and snowmelt elements that result in the development of high waters, particularly in July and August. Macedonian rivers are characterized by continental (complex) and Mediterranean (simple) discharge regimes. Except for these basic characteristics, the catchment area of the Kriva Reka has been spared major human influence because only three small dams have been built on it.

The analysis of high waters in the Kriva Reka basin was carried out at the Trnovec gauging station at 440 m above sea level, with a drainage area of 614.4 km^2. The total length of the river is 79 km, the minimum length is 50.6 km, the spring elevation is 1,590 m above sea level, and the confluence point elevation is 296 m above sea level. The total basin area is 1,001.8 km^2.

The total watershed length is 62.7 km, and the average basin width is determined at 16.0 km. The average basin elevation is 861.5 m above sea level. The river network density is 1.3 km/km^2, and the river network frequency is 0.5 streams/km^2. The Horton ratio (Horton 1932) is 0.16, which is lower than for other rivers (e.g., the Morava compactness coefficient is 0.37; Dukič 1984). A higher Horton ratio value means that the flood probability on the Kriva Reka is lower than on the Morava River. The Kriva Reka forest cover is 260 km^2 or 25.9%. The compactness coefficient is rather low (1.7), which means that the Kriva Reka basin does not form a circle and that the flood inflow from its tributaries is not simultaneous.

Figure 1: Upper course of the Kriva Reka.

2 Methods

This paper uses mathematical and statistical methods, such as tests of independence and homogeneity, covering a standard period in hydrological research. The maximum high waters were calculated for different return periods (from 2 to 10,000 years) and a graphic comparison and tests of correspondence between the empirical and theoretical distributions were performed. The theoretical high waters were calculated according to different statistical distributions.

Figure 4 clearly shows a trend of decreasing values, a wet period between 1961/62 and 1979/80, and a dry period between 1981/82 and 1999/2000.

Table 1: Annual maximum discharge (m^3/s) at the Trnovec gauging station from 1961/62 to 1999/2000. Data on the maximum annual discharges were obtained from the National Hydrometeorological Service in Skopje.

Year	Q_{max}	Year	Q_{max}	Year	Q_{max}	Year	Q_{max}
1961/62	89.8	1971/72	104.0	1981/82	49.0	1991/92	22.2
1962/63	108.0	1972/73	95.7	1982/83	24.9	1992/93	35.5
1963/64	249.0	1973/74	81.2	1983/84	31.3	1993/94	14.6
1964/65	106.0	1974/75	190.0	1984/85	33.5	1994/95	40.1
1965/66	264.0	1975/76	99.0	1985/86	24.4	1995/96	33.7
1966/67	54.4	1976/77	123.0	1986/87	45.2	1996/97	37.6
1967/68	175.0	1977/78	16.8	1987/88	12.2	1997/98	13.6
1968/69	85.4	1978/79	34.0	1988/89	13.9	1998/99	41.5
1969/70	313.0	1979/80	66.0	1989/90	12.2	1999/2000	15.3
1970/71	158.0	1980/81	33.5	1990/91	28.9		

Figure 2: Geographical position of the Kriva Reka drainage area in Macedonia.

3 Testing the independence and homogeneity of maximum annual discharge time series

When calculating the probability of occurrence of high waters at the Trnovec gauging station based on the series of maximum annual discharges, it is very important for the accuracy of the calculations that the series be independent and homogenous. The occurrences must be independent from one another and the series of maximum discharge must not be influenced by specific rare natural phenomena such as earthquakes, major landslides, major forest fires, and volcanic eruptions or human phenomena such as river regulation, construction, and large-scale logging (WMO 1994).

In order to statistically process the maximum discharges, a period of 30 years is necessary (Srebrenović 1986). If the period observed is shorter, the analysis is made using a shorter data period (Abida and Elluze 2008). In this case, a period of 39 years was processed.

3.1 Successive square method

At this stage of the analysis, we tested the independence of the maximum annual discharge data series for the Trnovec gauging station. In the successive square method, the statistic *u* (the value that represents the degree of independence) is calculated using the following equation (Shah 1970):

$$d^2 = \frac{1}{n-1}\sum_{i=1}^{n-1}\left(x_{i+1}-x_i\right)^2; \qquad (1)$$

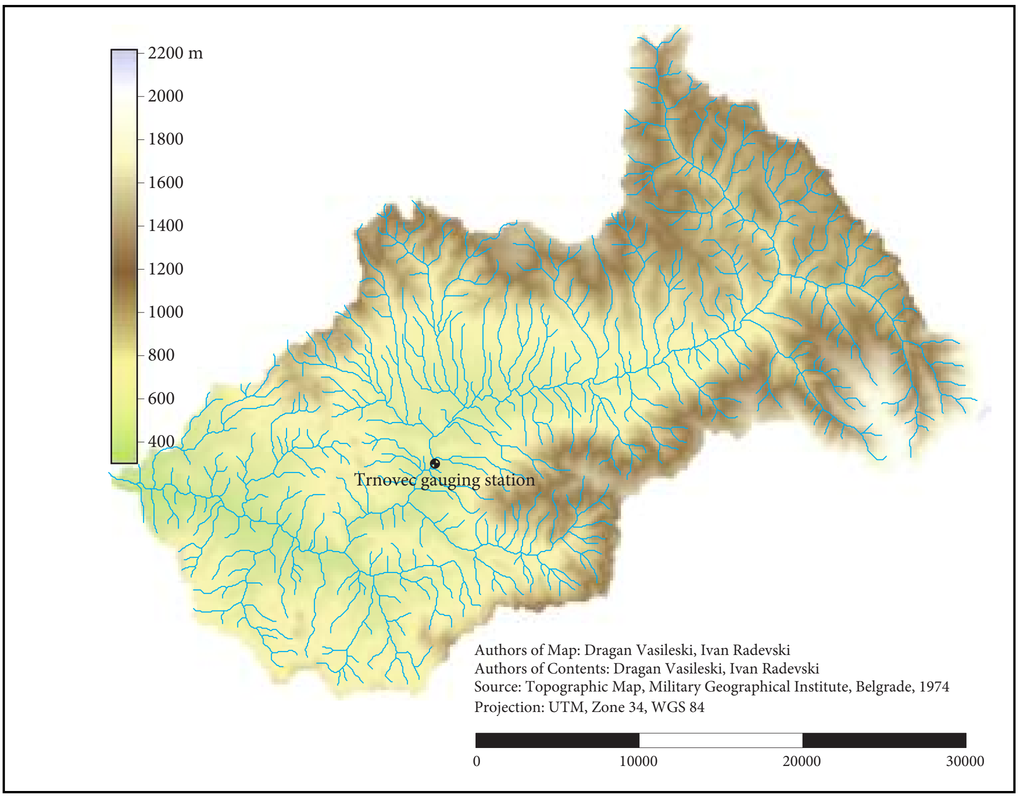

Figure 3: Location of the Trnovec gauging station in the Kriva Reka drainage area.

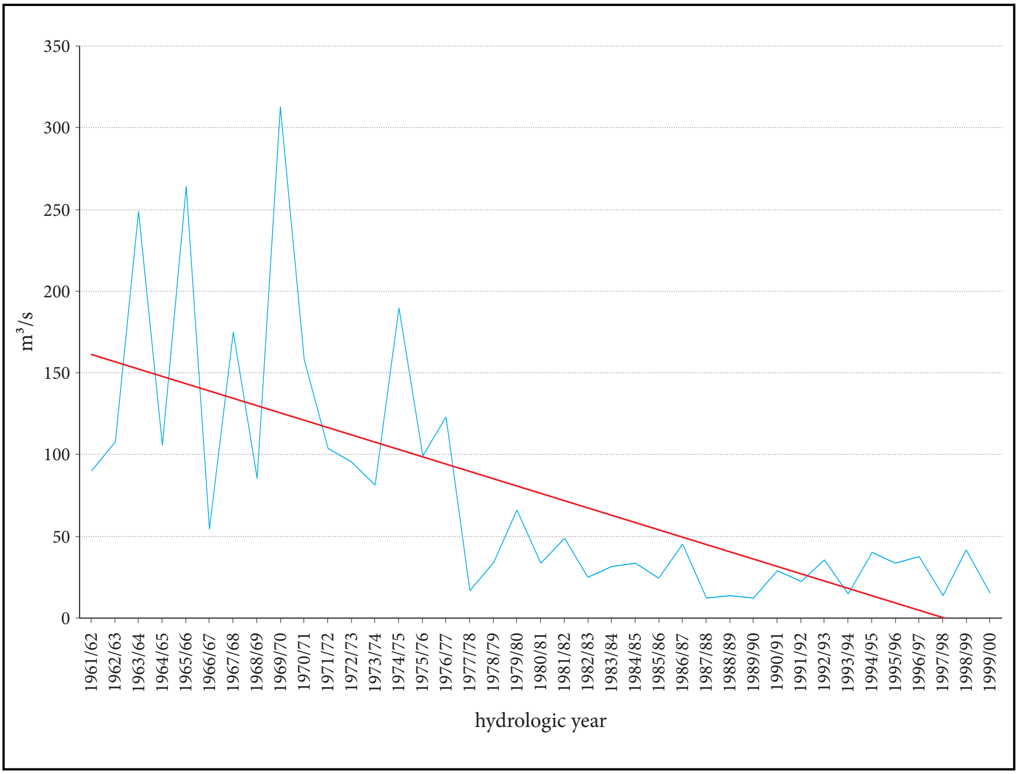

Figure 4: Time series of maximum annual discharges at the Trnovec gauging station from 1961/62 to 1999/2000.

In the above equation, x_i is annual maximum discharge and n is the number of members in the data series.

$$u = \frac{\dfrac{\frac{d^2}{2} - 1}{\sigma^2}}{\sqrt{\dfrac{n-2}{n^2-1}}} \qquad (2)$$

The main elements in the second equation are the variable d^2, σ (standard deviation) and the statistic u. The variable n is the number of data set members.

For the threshold of significance $\alpha = 0.05$, the statistic u is relevant if it is within the following limits: $-1.96 < u < 1.96$. The result $u = -2.593$ is not within the limits of $-1.96 < u < 1.96$, hence we can conclude that the series of maximum annual discharges is not independent; that is, its members are not independent from one another. This result confirms the non-randomness of maximum discharges in the period between 1961/62 and 1999/2000 at the Trnovec gauging station.

Testing has also proven the existence of a trend that represents a non-accidental component in statistics.

3.2 Kolmogorov–Smirnov test

The Kolmogorov–Smirnov test provides accurate results regarding the series' homogeneity (Popović and Blagojević 1997). In this particular case, the period of 39 hydrologic years was divided into two series (1961/62–1979/80 and 1980/81–1999/2000).

Table 2: Calculating the squares for the statistic *u*.

Hydrologic year	Q_{max} (m^3/s)	$x_{i-1} - x_i$	$(x_{i-1} - x_i)^2$
1961/62	89.8	*	*
1962/63	108.0	18.20	331.2
1963/64	249.0	141.00	19,881.0
1964/65	106.0	–143.00	20,449.0
1965/66	264.0	158.00	24,964.0
1966/67	54.4	–209.60	43,932.2
1967/68	175.0	120.60	14,544.4
1968/69	85.4	–89.60	8,028.2
1969/70	313.0	227.60	51,801.8
1970/71	158.0	–155.00	24,025.0
1971/72	104.0	–54.00	2,916.0
1972/73	95.7	–8.30	68.9
1973/74	81.2	–14.50	210.3
1974/75	190.0	108.80	11,837.4
1975/76	99.0	–91.00	8,281.0
1976/77	123.0	24.00	576.0
1977/78	16.8	–106.20	11,278.4
1978/79	34.0	17.20	295.8
1979/80	66.0	32.00	1,024.0
1980/81	33.5	–32.50	1,056.3
1981/82	49.0	15.50	240.3
1982/83	24.9	–24.10	580.8
1983/84	31.3	6.40	41.0
1984/85	33.5	2.20	4.8
1985/86	24.4	–9.10	82.8
1986/87	45.2	20.80	432.6
1987/88	12.2	–33.00	1,089.0
1988/89	13.9	1.70	2.9
1989/90	12.2	–1.70	2.9
1990/91	28.9	16.70	278.9
1991/92	22.2	–6.70	44.9
1992/93	35.5	13.30	176.9
1993/94	14.6	–20.90	436.8
1994/95	40.1	25.50	650.3
1995/96	33.7	–6.40	41.0
1996/97	37.6	3.86	14.9
1997/98	13.6	–23.94	573.3
1998/99	41.5	27.88	777.3
1999/2000	15.3	–26.19	686.0

The maximum difference *Dn* is 0.4. Considering the fact that for a determined threshold of significance $\alpha = 0.05$, which is standard in hydrologic studies, the maximum difference *Dn* must not exceed 0.21. Hence we can conclude that the series is not homogenous. The reasons for such a result could be either natural or manmade. The discharges during the second period have significantly lower values compared to the first period. This is most likely due to the three small dams built in the catchment area.

4 Empirical distribution of the annual maximum water discharge using the Weibull equation

When using mathematical statistics methods, it is very important to take into consideration several basic characteristics of the relation between empirical and theoretical distributions. Empirical distributions usually deviate from theoretical distributions. The deviation is larger in the case of small samples. In hydrology a standard sample is considered to be a period of 30 years, and a larger sample will certainly provide more

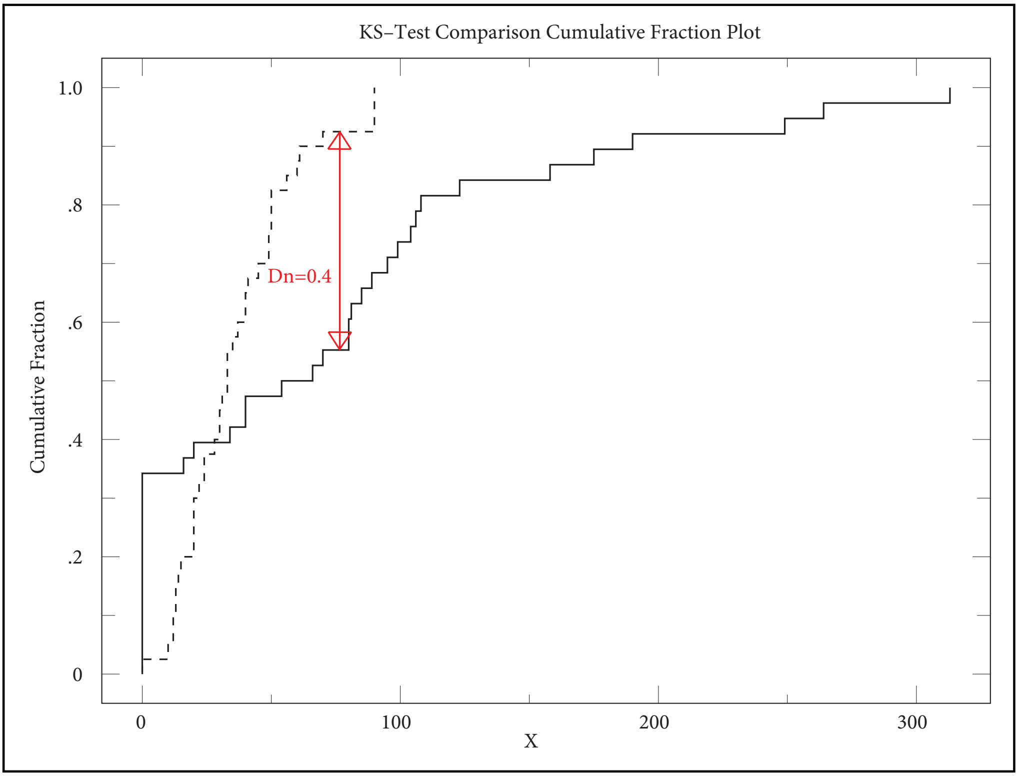

Figure 5: Kolmogorov–Smirnov test for homogeneity of maximum discharges.

Figure 6: The Kriva Reka near the Trnovec gauging station.

accurate results. Often several theoretical distributions are used, which display great differences in results; that is, small probabilities despite a satisfactory correspondence with the empirical distribution.

In addition to errors that occur as a result of short observation periods, errors also result from inaccuracy in measurements or a mistake in the assessment of parameters of theoretical calculations (Jovanović 1987).

The Weibull equation was used to determine the empirical distribution and the return periods of high discharges:

$$P_m = \frac{m}{N+1} \tag{3}$$

m is the rank in time series, *N* is the number of members in time series, and *Pm* is empirical probability expressed in percent. The Weibull equation was used to calculate the empirical distribution of maximum annual discharges for the period studied. The results from this equation are presented in Figures 7 and 8.

Table 3: Empirical distribution according to the Weibull equation for maximum annual discharges at the Trnovec gauging station from 1961/62 to 1999/2000.

m	Chronological order		Descending order		*P*= *m* / (*N*+1) * 100
1	1961/62	89.8	1969/70	313.0	2.50
2	1962/63	108.0	1965/66	264.0	5.00
3	1963/64	249.0	1963/64	249.0	7.50
4	1964/65	106.0	1974/75	190.0	10.00
5	1965/66	264.0	1967/68	175.0	12.50
6	1966/67	54.4	1970/71	158.0	15.00
7	1967/68	175.0	1976/77	123.0	17.50
8	1968/69	85.4	1962/63	108.0	20.00
9	1969/70	313.0	1964/65	106.0	22.50
10	1970/71	158.0	1971/72	104.0	25.00
11	1971/72	104.0	1975/76	99.0	27.50
12	1972/73	95.7	1972/73	95.7	30.00
13	1973/74	81.2	1961/62	89.8	32.50
14	1974/75	190.0	1968/69	85.4	35.00
15	1975/76	99.0	1973/74	81.2	37.50
16	1976/77	123.0	1979/80	66.0	40.00
17	1977/78	16.8	1966/67	54.4	42.50
18	1978/79	34.0	1981/82	49.0	45.00
19	1979/80	66.0	1986/87	45.2	47.50
20	1980/81	33.5	1998/99	41.5	50.00
21	1981/82	49.0	1994/95	40.1	52.50
22	1982/83	24.9	1996/97	37.6	55.00
23	1983/84	31.3	1992/93	35.5	57.50
24	1984/85	33.5	1978/79	34.0	60.00
25	1985/86	24.4	1995/96	33.7	62.50
26	1986/87	45.2	1984/85	33.5	65.00
27	1987/88	12.2	1980/81	33.5	67.50
28	1988/89	13.9	1983/84	31.3	70.00
29	1989/90	12.2	1990/91	28.9	72.50
30	1990/91	28.9	1982/83	24.9	75.00
31	1991/92	22.2	1985/86	24.4	77.50
32	1992/93	35.5	1991/92	22.2	80.00
33	1993/94	14.6	1977/78	16.8	82.50
34	1994/95	40.1	1999/2000	15.3	85.00
35	1995/96	33.7	1993/94	14.6	87.50
36	1996/97	37.6	1988/89	13.9	90.00
37	1997/98	13.6	1997/98	13.6	92.50
38	1998/99	41.5	1989/90	12.2	95.00
39	1999/2000	15.3	1987/88	12.2	97.50

In Table 3, maximum discharges have been arranged in chronological and descending order. The average maximum discharge for the period between 1961/62 and 1999/2000 is 76.33 m^3/s, and the standard deviation is 74.58. The variation coefficient is 0.98, and the coefficient of skewness is 1.69. These two coefficients are necessary for calculating the theoretical Pearson type III distributions.

5 Estimation of theoretical maximum discharges

The theoretical maximum discharges were estimated according to five distributions:
– Gaussian or normal distribution,
– Pearson type III distribution,
– Log-Pearson type III distribution,
– Gumbel distribution,
– Log-normal distribution.

5.1 Gaussian or normal distribution

The Gaussian distribution is the only symmetrical distribution of the five distributions used. Symmetry leads to lower values of maximum discharges at small probabilities. The basic parameters of this distribution are the arithmetic mean and the standard deviation (Srebrenović 1986).

Table 4: Theoretical maximum discharges for corresponding return periods using the Gaussian distribution.

T (years)	*P* (%)	*z*	$z \cdot \sigma$	Q_{max}
10,000	0.01	3.72	277.05	353.35
1,000	0.1	3.09	230.44	306.74
200	0.5	2.58	192.11	268.41
100	1	2.33	173.46	249.76
50	2	2.05	153.18	229.48
25	4	1.75	130.66	206.96
20	5	1.64	122.30	198.60
10	10	1.28	95.46	171.76
5	20	0.84	62.79	139.09
2	50	0.00	0.00	76.30

According to the Gaussian distribution, the following maximum discharges are expected at the Trnovec gauging station: 353.35 m^3/s once every 10,000 years, 306.74 m^3/s once every 1,000 years, and 249.76 m^3/s once every 100 years.

5.2 Pearson type III distribution

The Pearson type III distribution is frequently used to calculate the probability of occurrence of maximum discharges (Apolov 1963; Srebrenović 1986). This theoretical distribution corresponds well with the empirical distribution, especially when calculating maximum discharges with return periods of 10,000 or 1,000 years.

Table 5: Theoretical maximum discharges for corresponding return periods using the Pearson type III distribution.

T (years)	*P* (%)	φ	$\varphi \cdot C_v$	$K_s = \varphi\ C_v + 1$	$Q_{max} = Q_{авmax} \cdot K_s$
10,000	0.01	7.52	7.35	8.35	637.20
1,000	0.1	5.50	5.38	6.38	486.52
200	0.5	4.08	3.99	4.99	380.61
100	1	3.44	3.36	4.36	332.87
50	2	2.82	2.76	3.76	286.63
25	4	2.19	2.14	3.14	239.64
20	5	1.97	1.93	2.93	223.23
10	10	1.32	1.29	2.29	174.75
5	20	0.66	0.65	1.65	125.53
2	50	–0.27	–0.26	0.74	56.16

The theoretical maximum discharges for the corresponding return period were calculated using the following equation:

$$Q_{\max} = \left(C_v \cdot \varphi + 1\right) \cdot \bar{x} \quad (4)$$

The main parameters of this distribution are the arithmetic mean, the coefficient of variation, and the coefficient of skewness. The Pearson coefficient was obtained from the basic Pearson tables.

According to the Pearson type III distribution, the following maximum discharges are expected at the Trnovec gauging station: 637.20 m^3/s once every 10,000 years, 486.52 m^3/s once every 1,000 years, and 332.87 m^3/s once every 100 years.

5.3 Log-Pearson type III distribution

The basic parameters of this distribution are the average maximum discharge ($\bar{q}$), the coefficient of variation (*Cv*), and the coefficient of skewness (*Cs*). The theoretical maximum discharges for the corresponding return period were calculated using the following equations:

$$\bar{q} = \log Q_{\max} \quad (5)$$

$$\bar{q} = \frac{1}{2} \sum_{i=1}^{n} q \quad (6)$$

$$\bar{\sigma} = \pm \sqrt{\frac{\sum_{i=1}^{n} (q - q)^2}{n}} \quad (7)$$

Table 6: Theoretical maximum discharges for corresponding return periods using the log-Pearson type III distribution.

T (years)	*P* (%)	φ	*Cv* * φ + 1	$\bar{q}$	log Q_{max}	Q_{max}
10,000	0.01	4.20	1.99	1.70	3.38	2420.25
1,000	0.1	3.40	1.80	1.70	3.06	1157.97
200	0.5	2.80	1.66	1.70	2.82	666.15
100	1	2.50	1.59	1.70	2.70	505.26
50	2	2.18	1.51	1.70	2.58	376.22
25	4	1.82	1.43	1.70	2.43	270.01
20	5	1.71	1.40	1.70	2.39	243.30
10	10	1.31	1.31	1.70	2.23	169.07
5	20	0.82	1.19	1.70	2.03	107.44
2	50	–0.07	0.98	1.70	1.68	47.31

According to the log-Pearson type III distribution, the following maximum discharges are expected at the Trnovec gauging station: 2,420.25 m^3/s once every 10,000 years, 1,157.97 m^3/s once every 1,000 years, and 505.26 m^3/s once every 100 years.

5.4 Gumbel distribution

The basic parameters of this distribution are the average maximum discharge and the standard deviation (Gumbel 1958). The theoretical maximum discharges for the corresponding return period were calculated using the following equations:

$$\frac{1}{\alpha} = 0{,}78 \cdot \sigma \quad (8)$$

$$Q_{\mathrm{m}} = \bar{x} - 0{,}577 \cdot \frac{1}{\alpha} \quad (9)$$

$$Q_{\max} = Q_m + z \cdot Q_{\max} = Q_m + z \cdot \frac{1}{\alpha} \quad (10)$$

Table 7: Theoretical maximum discharges for corresponding return periods using the Gumbel distribution.

T (years)	*P* (%)	*z*	$z \cdot 1/\alpha$	Q_m	Q_{max}
1,0000	0.01	9.21	535.74	42.74	578.47
1,000	0.1	6.91	401.95	42.74	444.68
200	0.5	5.30	308.18	42.74	350.92
100	1	4.60	267.58	42.74	310.31
50	2	3.91	227.44	42.74	270.18
25	4	3.20	186.14	42.74	228.88
20	5	2.97	172.76	42.74	215.50
10	10	2.27	132.04	42.74	174.78
5	20	1.50	87.25	42.74	129.99
2	50	0.37	21.35	42.74	64.08

According to the Gumbel distribution, the following maximum discharges are expected at the Trnovec gauging station: 578.47 m^3/s once every 10,000 years, 444.68 m^3/s once every 1,000 years, and 310.31 m^3/s once every 100 years.

5.5 Log-normal distribution

The basic parameters of this distribution are the average logarithmic values of maximum discharges and the standard deviation of the logarithmic values of maximum discharges. The theoretical maximum discharges for the corresponding return period were calculated using the following equations:

$$Q_{max} = \bar{q} + z + \bar{\sigma} \tag{11}$$

$$Q_{max} = 10^{q_{max}} \tag{12}$$

Table 8: Theoretical maximum discharges for corresponding return periods using the log-normal distribution.

T (years)	*P* (%)	*z*	$z \cdot \sigma$	$\bar{q}$	$\log Q_{max}$	Q_{max}
10,000	0.01	3.72	1.49	1.70	3.19	1545.25
1,000	0.1	3.09	1.24	1.70	2.94	868.96
200	0.5	2.58	1.03	1.70	2.73	541.25
100	1	2.33	0.93	1.70	2.63	429.93
50	2	2.05	0.82	1.70	2.53	334.66
25	4	1.75	0.70	1.70	2.40	253.40
20	5	1.64	0.66	1.70	2.36	228.56
10	10	1.28	0.51	1.70	2.22	164.06
5	20	0.84	0.34	1.70	2.04	109.60
2	50	0.00	0.00	1.70	1.70	50.47

According to the log-normal distribution, the following maximum discharges are expected at the Trnovec gauging station: 1,545.25 m^3/s once every 10,000 years, 868.96 m^3/s once every 1,000 years, and 429.93 m^3/s once every 100 years.

6 Conclusion

The series of a 30-year period is considered to be a standard measurement unit in hydrology. Even though the data series was sufficiently long, the series' testing has disproved its independence, homogeneity, and representativity. The analysis showed a decreasing trend from the beginning to the end of the period studied. To be more precise, discharges of approximately 100 m^3/s and above occur only in the first half of the series.

The main purpose of this research was to provide findings that would facilitate flood prevention.

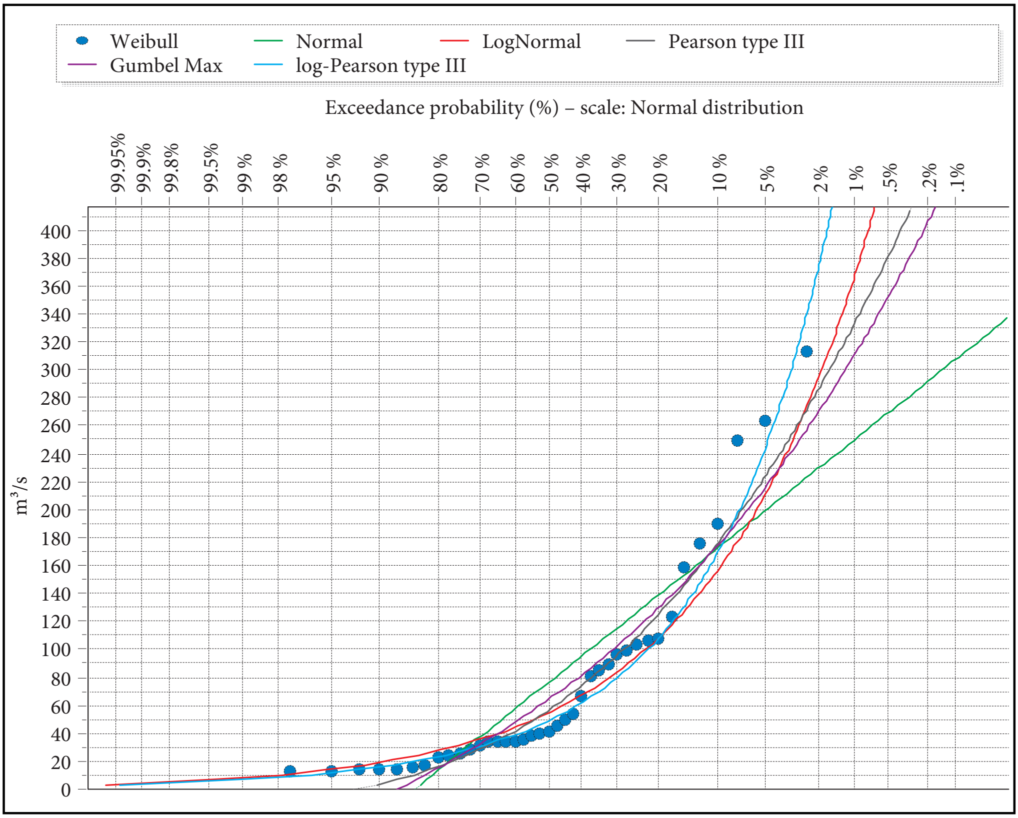

Figure 7: Probability plot with five cumulative frequency distributions compared with the Weibull empirical distribution.

A comparison with the Kolmogorov–Smirnov test was made in order to see which of the five theoretical distributions used match the Weibull empirical distribution for the determined threshold of significance $\alpha = 0.05$.

The probability plot (Figure 7) shows a satisfactory correspondence between the Weibull empirical distribution and the log-Pearson type III distribution (the blue dotted line), especially for high discharge values. There is also a satisfactory correspondence between the empirical distribution of maximum annual discharges and the log-normal distribution, which is not the case with other distributions. The correspondence between the logarithmic distributions and the empirical distribution is common in streams subject to flash floods. Therefore, flood protection should be based on logarithmic distributions.

Figure 8 clearly shows a correspondence between the log-Pearson type III distribution and the empirical distribution, and a 95% confidence interval.

Table 9: Kolmogorov–Smirnov test results.

K–S test	$\alpha = 1\%$	$\alpha = 5\%$	$\alpha = 10\%$	attained α	*Dn*
Normal	ACCEPT	ACCEPT	REJECT	9.89%	0.1928
Log-normal	ACCEPT	ACCEPT	ACCEPT	48.12%	0.1309
Pearson type III	ACCEPT	ACCEPT	ACCEPT	36.58%	0.1438
Log-Pearson type III	ACCEPT	ACCEPT	ACCEPT	94.18%	0.0813
Gumbel	ACCEPT	ACCEPT	ACCEPT	25.08%	0.1596

According to the results of the Kolmogorov–Smirnov test (Table 9), four distributions are accepted, except for the normal distribution, which is rejected at the significance level $\alpha = 10\%$. The best correspondence

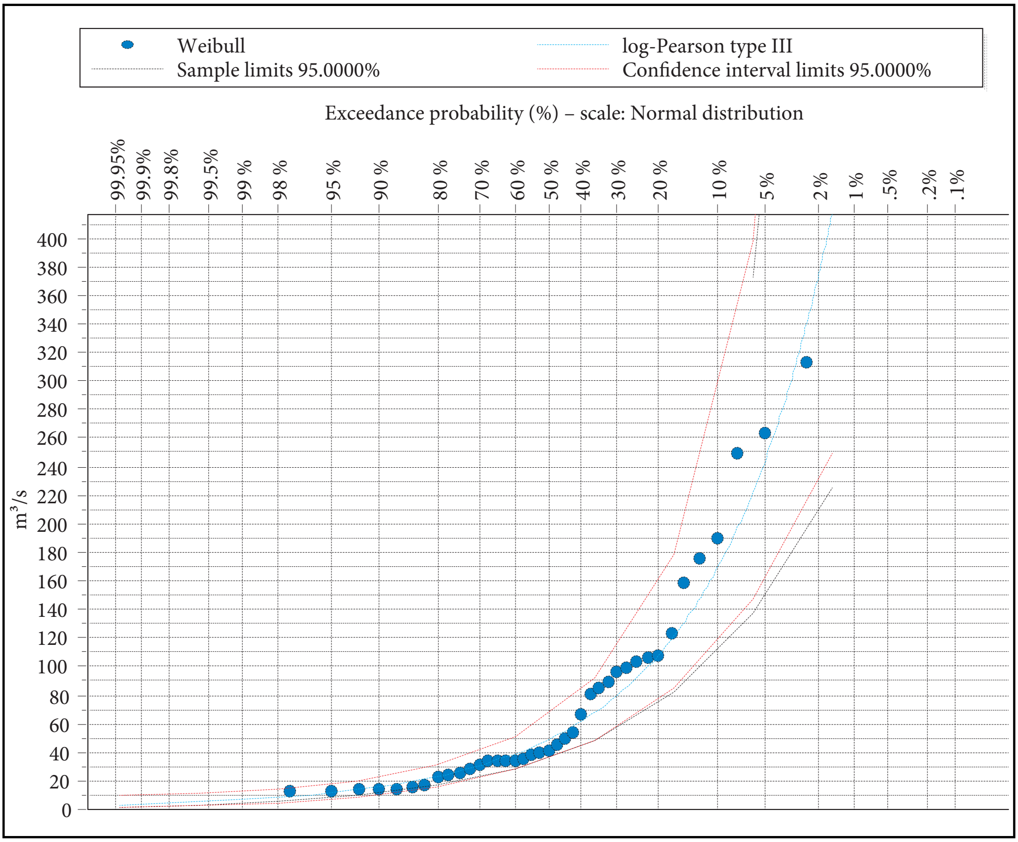

Figure 8: The log-Pearson type III distribution 95% confidence interval limits using a Monte Carlo simulation.

is achieved by the log-Pearson type III distribution, which shows the smallest maximum difference between the empirical and the theoretical distribution ($Dn = 0.0813$). The results confirm the high fluctuation of water discharges of the Kriva Reka.

Table 10: Chi-squared test results.

Chi-squared test	α = 1 %	α = 5 %	α = 10 %	attained α	Pearson parameter.
Normal	REJECT	REJECT	REJECT	0.00%	29.3333
Log-normal	ACCEPT	ACCEPT	ACCEPT	20.40%	3.17949
Pearson type III	ACCEPT	REJECT	REJECT	1.92%	5.48718
Log-Pearson type III	ACCEPT	ACCEPT	ACCEPT	16.84%	1.89744
Gumbel	REJECT	REJECT	REJECT	0.26%	11.8974

According to the chi-squared test, the log-Pearson type III distribution best corresponds to the empirical distribution. In addition to this distribution, the only accepted distribution for the three significance levels is the log-normal distribution.

In the analysis of the probability of occurrence of high discharges in western Macedonia, the log-Pearson type III distribution shows the best correspondence for the Radika River (Vasileski 1993), and the Gumbel distribution shows the best correspondence for the Crna Reka River (Radevski 2010), which indicates a smaller annual fluctuation of maximum discharges.

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